SCHOLAR Study Guide

National 5 Mathematics

Course Materials
Topic 9: Factorising

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Topic 9

Factorising

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Learning objective

By the end of this topic, you should be able to:

- factorise using a single common factor;
- factorise a difference of two squares;
- factorise trinomials.

9.1 Looking back at National 4: Expressions with a Common Factor

Expressions using a Common Factor Go online

A common factor has been used to factorise an expression:

$$\begin{aligned} 3(x+4) &= 3x + 3 \times 4 \\ &= 3x + 12 \end{aligned}$$

So, $3x + 12 = 3(x + 4)$.

Practise spotting the common factor and adding the brackets...

What was the expression before we got rid of the brackets?

What was $45x + 10$?

Both terms can be divided by 5... So it used to be $5(9x + 2)$.

Check it: $5 \times 9x + 5 \times 2 = 45x + 10$

Example

Problem:

Factorise $7a + 35$.

Solution:

Note that 7 is the largest number that goes into both 7 and 35.

$\Rightarrow 7a + 35 = 7(a + 5)$

You can check your answer by multiplying out the brackets, you should end up with the original expression.

$7(a + 5) = 7 \times a + 7 \times 5 = 7a + 35$

Q1:

Factorise $8y + 40$.

9.2 Factorising using a simple common factor

In *Topic 8: Expanding brackets*, you started with an expression containing brackets and removed them. In this topic you will start with an expression without brackets and add brackets in order to factorise it.

Simple common factors Go online

The following example shows you how to factorise an expression by identifying a common factor.

Factorising is the inverse of removing brackets.....

$$15e + 10 \rightleftharpoons 5(3e + 2)$$

$$12n + 8 \rightleftharpoons 4(3n + 2)$$

.... multiply out the brackets to check.

Examples

1.

Problem:

Factorise $3x - 3$

Solution:

The terms $3x$ and -3 have a common factor of 3.

$$\begin{aligned} & 3x - 3 \\ = \ & 3 \times x - 3 \times 1 \\ = \ & 3(x - 1) \end{aligned}$$

.....

2.

Problem:

Factorise $12x + 18$

Solution:

The terms $12x$ and $+18$ have common factors of 2 and 6. Since 6 is the highest common factor we must use it.

$$\begin{aligned} & 12x + 18 \\ = \ & 6 \times 2x + 6 \times 3 \\ = \ & 6(2x + 3) \end{aligned}$$

.....

3.

Problem:

Factorise $x^2 + x$

Solution:

The terms x^2 and x have a common factor of x.

$$\begin{aligned} & x^2 + x \\ = \ & x \times x + x \times 1 \\ = \ & x(x + 1) \end{aligned}$$

.....

4.

Problem:

Factorise $4x^2 + 20x$

Solution:

The terms $4x^2$ and $20x$ have common factors of $2x$ and $4x$.

$$\begin{aligned} & 4x^2 + 20x \\ = & \ 4x \times x + 4x \times 5 \\ = & \ 4x(x + 5) \end{aligned}$$

Factorising using simple common factors practice

Go online

Q2: Factorise $8a + 42$

.....

Q3: Factorise $10b - 45$

.....

Q4: Factorise $c^2 + 22c$

.....

Q5: Factorise $4d^2 - 28d$

.....

Q6: Factorise $3g^2h + 9gh^2$

.....

Q7: Factorise $e^3 + 3e^2 - 5e$

Factorising using a simple common factor exercise

Go online

These questions are for practice only.

Q8: Factorise $6x - 12$

.....

Q9: Factorise $24 + 32z$

.....

Q10: Factorise $y^2 - 8y$

.....

Q11: Factorise $9a - 6b + 3c$

.....

Q12: Factorise $\pi r^2 + 2\pi r$

. . . . . .

Q13: Factorise $2d^3e - 6de^2$

9.3 Factorising a difference of two squares

In *9.1: Factorising using a simple common factor*, you started with an expression without brackets and added them in order to factorise it. In this sub-topic you will now practise doing this with the special case - the difference of two squares.

Factorising: Difference of two squares Go online

The following example shows how to factorise the difference of two squares.

Look out for the difference of two squares.....

$25g^2 - 25h^2 \rightleftharpoons (5g - 5h)(5g + 5h)$

$16g^2 - d^2 \rightleftharpoons (4g - d)(4g + d)$

$9g^2 - 25p^2 \rightleftharpoons (3g - 5p)(3g + 5p)$

.... multiply out the brackets to check.

Examples

1.

Problem:

Factorise $x^2 - g^2$

Solution:

We have the difference of two squares so $x^2 - g^2 = (x - g)(x + g)$

. . . . . .

2.

Problem:

Factorise $9x^2 - 1$

Solution:

We have $9x^2 - 1 = 3^2x^2 - 1^2$ which is the difference of two squares so,
$9x^2 - 1 = (3x - 1)(3x + 1)$

. . . . . .

3.

Problem:

Factorise $49x^2 - 16$

Solution:

We have $49x^2 - 16 = 7^2x^2 - 4^2$ which is a difference of two squares,
$49x^2 - 16 = (7x - 4)(7x + 4)$

.....

4.

Problem:

Factorise $50a^2 - 18$

Solution:

We have a simple common factor of 2 giving $50a^2 - 18 = 2(25a^2 - 9)$
$25a^2 - 9$ is a difference of two squares so $2(25a^2 - 9) = 2(5^2a^2 - 3^2)$
hence $50a^2 - 18 = 2(5a - 3)(5a + 3)$

.....

5.

Problem:

Factorise $x^2 + y^2$

Solution:

We have the SUM of two squares. This does **not** factorise.

Factorising a difference of two squares practice Go online

Q14: Factorise $a^2 - b^2$

.....

Q15: Factorise $36c^2 - d^2$

.....

Q16: Factorise $e^2 - 81$

.....

Q17: Factorise $100f^2 - 25$

.....

Q18: Factorise $16g^2 - 49h^2$

.....

Q19: Factorise $18x^2 - 8$

.....

Q20: Factorise $27y^2 - 12z^2$

Factorising a difference of two squares exercise Go online

These questions are for practice only.

Q21: Factorise $p^2 - q^2$

.....

Q22: Factorise $36a^2 - 121$

.....

Q23: Factorise $16p^2 - 81q^2$

.....

Q24: Factorise $12b^2 - 48$

.....

Q25: Factorise $28c^2 - 63d^2$

9.4 Factorising a trinomial

In this sub-topic you will start with a trinomial expression and add brackets in order to factorise it.

A trinomial expression has three terms. Trinomial expressions are normally called trinomials. Here are examples of some trinomials,

- $x^2 + 2x + 3$
- $3a^2 + 4a + 1$
- $4k^2 - 2k - 2$
- $2m^2 + 7m - 15$

Factorising a trinomial Go online

The following example shows you how to factorise a trinomial expression.

The factorised form of $x^2 + 11x + 30$ is $(x + a)(x + b)$

where $a \times b = +30$ and $a + b = +11$.

1) Begin by finding the factors of +30

2) Find the pair which add to make +11....

1)	2)
(-1) and (-30)	(-1) + (-30) = -31
(-2) and (-15)	(-2) + (-15) = -17
(-3) and (-10)	(-3) + (-10) = -13
(-5) and (-6)	(-5) + (-6) = -11
1 and 30	1 + 30 = 31
2 and 15	2 + 15 = 17
3 and 10	3 + 10 = 13
5 and 6	5 + 6 = 11
6 and 5	**6 + 5 = 11**
10 and 3	10 + 3 = 13
15 and 2	15 + 2 = 17
30 and 1	30 + 1 = 31

We have $+6 + 5 = +11$ and $+6 \times +5 = +30$

This gives us values for ‘a’ and ‘b’ of +6 and +5.

So we can write... $x^2 + 11x + 30 = (x + 6)(x + 5)$

Multiply out the brackets to check.

Examples

1.

Problem:

What two numbers add to make 9 and multiply to make 18?

Solution:

Go through the factor pairs that make 18 and see which pair adds to 9.

1×18 (sum = 19);
2×9 (sum = 11);
3×6 (sum = 9) . . . we have it!

The numbers are 3 and 6.

. . . . . .

2.

Problem:

What two numbers add to make -1 and multiply to make -12?

Solution:

Go through the factor pairs that make -12 and see which pair adds to -1.

-1×12 (sum = 11);
-2×6 (sum = 4);
-3×4 (sum = 1) . . . we nearly have it; the sign is wrong so change the signs;
3×-4 (sum = -1) . . . and we have it.

The numbers are 3 and -4.

. . . . . .

3.

Problem:

Factorise $x^2 + 9x + 18$

Solution:

We want two numbers which add to make 9 and multiply to make 18.

Go through the factor pairs that make 18 and see which pair adds to 9.
1×18 (sum = 19);
2×9 (sum = 11);
3×6 (sum = 9) . . . we have it!

The numbers are 3 and 6.

So we have $x^2 + 9x + 18 = (x + 3)(x + 6)$

. . . . . .

4.

Problem:

Factorise $x^2 - x - 12$

Solution:

We want two numbers which add to make -1 and multiply to make -12.

-1×12 (sum = 11);
-2×6 (sum = 4);
-3×4 (sum = 1) ... we nearly have it; the sign is wrong so change the signs ... 3×-4 (sum = -1).

The numbers are 3 and -4.
So we have $x^2 - x - 12 = (x-4)(x+3)$.

.....

5.

Problem:

Factorise $2y^2 + 5y + 2$

Solution:

As we now have $2y^2$ finding the factorised form requires more thought. We want two terms which multiply to make $2y^2$

$2y \times y = 2y^2$ so our brackets start $(2y \quad)(y \quad)$

Now we want two numbers which multiply to make +2, i.e. 2×1 or 1×2.

In this example we need to check that the sum of the products of the inner and outer terms gives us the middle term $5y$.

Multiplying the inner terms gives $2y$ and multiplying the outer terms gives $2y$ (sum $4y$) but the middle term we want is 5y so...

Multiplying the inner terms gives $1y$ and multiplying the outer terms $4y$ (sum $5y$).

This gives us the middle term we want. So we have $2y^2 + 5y + 2 = (2y+1)(y+2)$

.....

6.

Problem:

Factorise $14g^2 - 20g + 6$

Solution:

We should always check for a simple common factor first. This question has a common factor of 2 giving $2(7g^2 - 10g + 3)$

Next we want two terms to make $7g^2 7g \times g = 7g^2$ so our brackets start $(7g \quad)(g \quad)$

Now we want two numbers which multiply to make +3 i.e. 3×1 or 1×3.

In this example we need to check that the sum of the products of the inner and outer terms gives the middle term $-10g$.

$(7g+3)(g+1)$ makes the product of the inner terms $3g$ and the outer terms $7g$ (sum $10g$) we nearly have it but we wanted $-10g$...

the sign is wrong but we know that -3×-1 also makes 3... $(7g-3)(g-1) = 7g^2 - 10g + 3$

Hence $14g^2 - 20g + 6 = 2\left(7g^2 - 10g + 3\right) = 2(7g-3)(g-1)$

Factorising a trinomial practice Go online

Q26: What two numbers add to make 9 and multiply to make 20?

.....

Q27: What two numbers add to make 2 and multiply to make -15?

.....

Q28: Factorise $x^2 + 9x + 20$

.....

Q29: Factorise $x^2 + 6x - 16$

.....

Q30: Factorise $a^2 + 9a + 8$

.....

Q31: Factorise $b^2 - 6b + 5$

.....

Q32: Factorise $c^2 + c - 6$

.....

Q33: Factorise $d^2 - 3d - 10$

.....

Q34: Factorise $3g^2 + 4g + 1$

.....

Q35: Factorise $5h^2 + 3h - 2$

.....

Q36: Factorise $2j^2 + 2j - 12$

.....

Q37: Factorise $4k^2 - 2k - 2$

Factorising a trinomial exercise Go online

These questions are for practice only.

Q38: Factorise $x^2 + 11x + 18$

.....

Q39: Factorise $x^2 - 2x - 15$

.....

Q40: Factorise $a^2 - a - 30$

.....

Q41: Factorise $b^2 + 3b - 28$

.....

Q42: Factorise $2d^2 + 5d + 3$

.....

Q43: Factorise $3e^2 - 4e + 1$

.....

Q44: Factorise $6f^2 - 17f + 12$

.....

Q45: Factorise $2g^2 + 4g + 2$

.....

Q46: Factorise $4h^2 + 6h - 10$

9.5 Learning points

When factorising always ask yourself three questions:

1. Is there a simple common factor?
2. Is it a difference of two squares?
3. Is it a trinomial?

and remember you could have a simple common factor and a difference of two squares **or** a simple common factor and a trinomial.

9.6 End of topic test

End of topic 9 test Go online

Q47:

a) Factorise $3x - 6$
b) Factorise $y^2 + 8y$
c) Factorise $27a - 18b + 63c$
d) Factorise $p^2 - q^2$
e) Factorise $36p^2 - 25q^2$
f) Factorise $\pi r^3 + \pi r^2$
g) Factorise $10b^3c - 25bc^2$

.....

Q48:

a) Factorise $x^2 + 5x + 4$
b) Factorise $x^2 + 8x + 15$
c) Factorise $3x^2 + 27x + 60$
d) Factorise $2x^2 - 8x - 42$
e) Factorise $2x^2 + 7x - 15$
f) Factorise $5x^2 - 16x + 3$
g) Factorise $4x^2 - 14x + 6$

Answers to questions and activities

Topic 9: Factorising

Answers from page 3.

Q1: $8(y + 5)$

Factorising using simple common factors practice (page 5)

Q2: $2(4a + 21)$

Q3: $5(2b - 9)$

Q4: $c(c + 22)$

Q5: $4d(d - 7)$

Q6: $3gh(g + 3h)$

Q7: $e(e^2 + 3e - 5)$

Factorising using a simple common factor exercise (page 5)

Q8: $6(x - 2)$

Q9: $8(3 + 4z)$

Q10: $y\,(y - 8)$

Q11: $3(3a - 2b + c)$

Q12: $\pi r(r + 2)$

Q13: $2de(d^2 - 3e)$

Factorising a difference of two squares practice (page 7)

Q14: $a^2 - b^2 = (a - b)(a + b)$

Q15: $36c^2 - d^2 = (6c - d)(6c + d)$

Q16: $e^2 - 81 = (e - 9)(e + 9)$

Q17: $100f^2 - 25 = (10f - 5)(10f + 5)$

Q18: $16g^2 - 49h^2 = (4g - 7h)(4g + 7h)$

Q19: $18x^2 - 8 = 2(9x^2 - 4) = 2(3x - 2)(3x + 2)$

Q20: $27y^2 - 12z^2 = 3(9y^2 - 4z^2) = 3(3y - 2z)(3y + 2z)$

Factorising a difference of two squares exercise (page 8)

Q21: $(p-q)(p+q)$This is a special case - we have the difference of two squares.

Q22: $(6a-11)(6a+11)$

Q23: $(4p-9q)(4p+9q)$This is a special case - we have the difference of two squares.

Q24: $3(2b-4)(2b+4)$

Q25: $7(2c-3d)(2c+3d)$

Factorising a trinomial practice (page 12)

Q26: The numbers are 5 and 4.
$4 + 5 = 9$ and $4 \times 5 = 20$

Q27: The numbers are -3 and 5.
$-3 + 5 = 2$ and $(-3) \times 5 = -15$

Q28:

Hints:

- To check whether your answer is correct, multiply out the brackets and you should get the original trinomial.

Answer: $x^2 + 9x + 20 = (x+4)(x+5)$

Q29: $x^2 + 6x - 16 = (x-2)(x+8)$

Q30: $a^2 + 9a + 8 = (a+1)(a+8)$

Q31: $b^2 - 6b + 5 = (b-1)(b-5)$

Q32: $c^2 + c - 6 = (c+3)(c-2)$

Q33: $d^2 - 3d - 10 = (d-5)(d+2)$

Q34:

Hints:

-
- The sum of the products of the inner and outer terms gives $1g + 3g = 4g$.

Answer: $3g^2 + 4g + 1 = (3g+1)(g+1)$

Q35:

Hints:

- The sum of the products of the inner and outer terms gives $-2h + 5h = 3h$.

Answer: $5h^2 + 3h - 2 = (5h-2)(h+1)$

Q36:

Hints:

- The simple common factor is 2 giving $2(j^2 + j - 6)$ and $j^2 + j - 6 = (j + 3)(j - 2)$.

Answer: $2j^2 + 2j - 12 = 2(j + 3)(j - 2)$

Q37:

Hints:

- The simple common factor is 2 giving $2(2k^2 - k - 1)$ and $2k^2 - k - 1 = (2k + 1)(k - 1)$.
- Remember the sum of the product of the inner and outer terms gives $1k + (-2k) = -k$

Answer: $4k^2 - 2k - 2 = 2(2k + 1)(k - 1)$

Factorising a trinomial exercise (page 13)

Q38: $(x + 2)(x + 9)$

Q39: $(x - 5)(x + 3)$

Q40: $(a - 6)(a + 5)$

Q41: $(b + 7)(b - 4)$

Q42: $(2d + 3)(d + 1)$

Q43: $(3e - 1)(e - 1)$

Q44: $(3f - 4)(2f - 3)$

Q45: $2(g + 1)(g + 1)$

Q46: $2(2h + 5)(h - 1)$

End of topic 9 test (page 15)

Q47:

a) $3(x - 2)$
b) $y(y + 8)$
c) $9(3a - 2b + 7c)$
d) $(p - q)(p + q)$
e) $(6p - 5q)(6p + 5q)$
f) $\pi r^2(r + 1)$
g) $5bc(2b^2 - 5c)$

Q48:

a) $(x+4)(x+1)$
b) $(x+3)(x+5)$
c) $3(x+4)(x+5)$
d) $2(x-7)(x+3)$
e) $(2x-3)(x+5)$
f) $(5x-1)(x-3)$
g) $2(2x-1)(x-3)$