SCHOLAR Study Guide

National 5 Mathematics

Course Materials
Topic 4: Volume of solids

Authored by:

Margaret Ferguson

Reviewed by:

Jillian Hornby

Previously authored by:

Eddie Mullan

Heriot-Watt University

Edinburgh EH14 4AS, United Kingdom.

First published 2014 by Heriot-Watt University.

This edition published in 2018 by Heriot-Watt University SCHOLAR.

Copyright © 2018 SCHOLAR Forum.

Members of the SCHOLAR Forum may reproduce this publication in whole or in part for educational purposes within their establishment providing that no profit accrues at any stage, Any other use of the materials is governed by the general copyright statement that follows.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, without written permission from the publisher.

Heriot-Watt University accepts no responsibility or liability whatsoever with regard to the information contained in this study guide.

Distributed by the SCHOLAR Forum.

SCHOLAR Study Guide Course Materials Topic 4: National 5 Mathematics

1. National 5 Mathematics Course Code: C847 75

Acknowledgements

Thanks are due to the members of Heriot-Watt University's SCHOLAR team who planned and created these materials, and to the many colleagues who reviewed the content.

We would like to acknowledge the assistance of the education authorities, colleges, teachers and students who contributed to the SCHOLAR programme and who evaluated these materials.

Grateful acknowledgement is made for permission to use the following material in the SCHOLAR programme:

The Scottish Qualifications Authority for permission to use Past Papers assessments.

The Scottish Government for financial support.

The content of this Study Guide is aligned to the Scottish Qualifications Authority (SQA) curriculum.

All brand names, product names, logos and related devices are used for identification purposes only and are trademarks, registered trademarks or service marks of their respective holders.

Topic 4

Volumes of solids

Contents

Learning objective

By the end of this topic, you should be able to:

- calculate the volume of a sphere, cone, pyramid and prism;
- calculate an unknown dimension given the volume of a solid;
- calculate the volume of composite solids.

© HERIOT-WATT UNIVERSITY

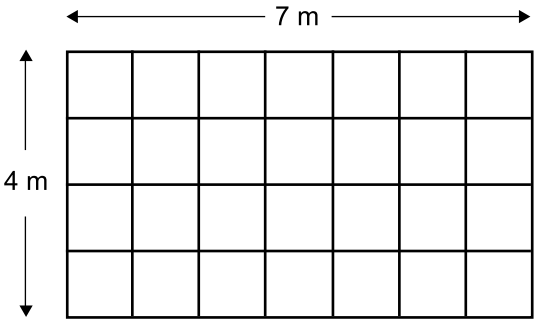
4.1 Looking back at National 4: Areas and volumes

Here we will look back at how to find the area of composite shapes and also how to find the volume of a cube and cuboid.

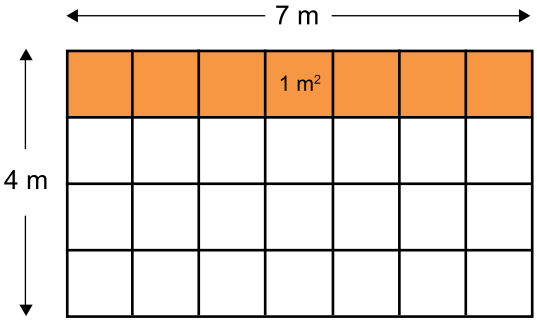
4.1.1 Areas of composite shapes

Area of a rectangle Go online

A floor is 4 m long by 7 m wide.



A carpet layer rolls out one length of carpet which is a metre wide. One length gives an area of 7 m^2.



Four lengths gives an area of $4 \times 7 = 28$ m^2.

Area of a rectangle = Length $\times$ Breadth

Example

Problem:

Find the area of a rectangle which is 3 cm long by 1·5 cm wide.

Solution:

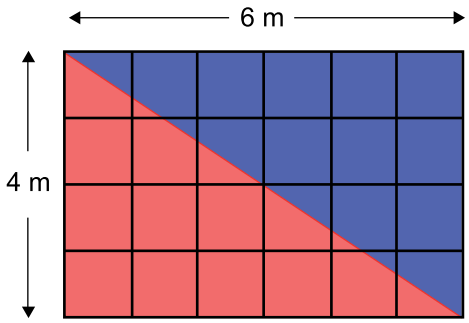
A = LB

$A = 3 \times 1 \cdot 5$

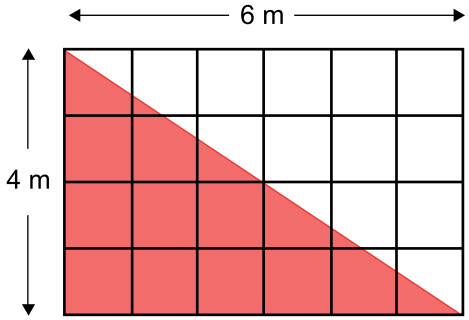
$A = 4 \cdot 5$ cm^2 [Note the units . . . we're counting squares]

Area of a right angled triangle

Go online



A 4 m by 6 m floor is carpeted as shown using red and blue carpet tiles that are each 1 m^2. What is the area of the red section?



Parts of squares are red. . . they are not easily counted. . . Until we realise that the red triangle is **half** of the rectangle.

Area of a triangle = $^1/_2$ of rectangle

Area of a triangle = $^1/_2$ of 4×6
Area of a triangle = $^1/_2$ of 24
Area of a triangle = 12 m^2.

Key point

Every right angled triangle is half of a rectangle.

Area of a triangle = $^1/_2 \times$ base $\times$ height

Example

Problem:

Find the area of a right angled triangle which is has a base 3 cm long and an altitude of 1·5 cm.

© HERIOT-WATT UNIVERSITY

Solution:

$A = {}^1\!/_2\, base \times altitude$
$A = {}^1\!/_2 \times 3 \times 1 \cdot 5$
$A = {}^1\!/_2 \times 4 \cdot 5$
$A = 2 \cdot 25$ cm^2 [Note the units . . . we're counting squares]

Key point

The formulae for the areas of rectangles and triangles are:

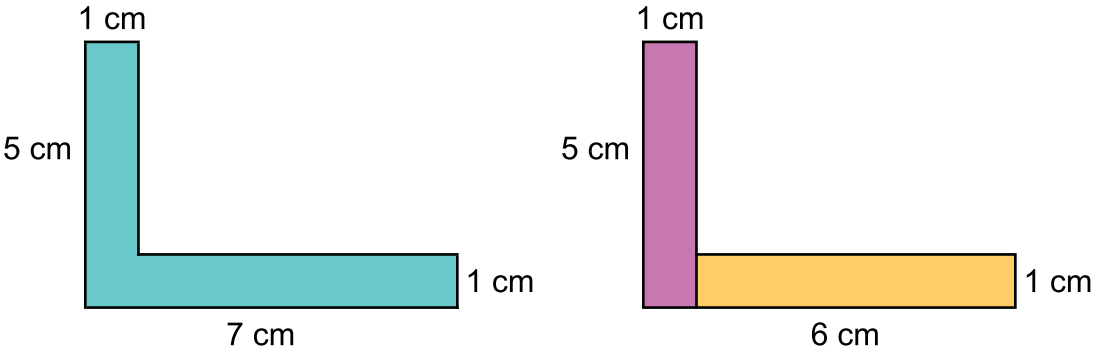
Area of a rectangle $= l \times b$ where l is the length and b is the breadth.

Area of a triangle = ${}^1\!/_2 \times$ base $\times$ height

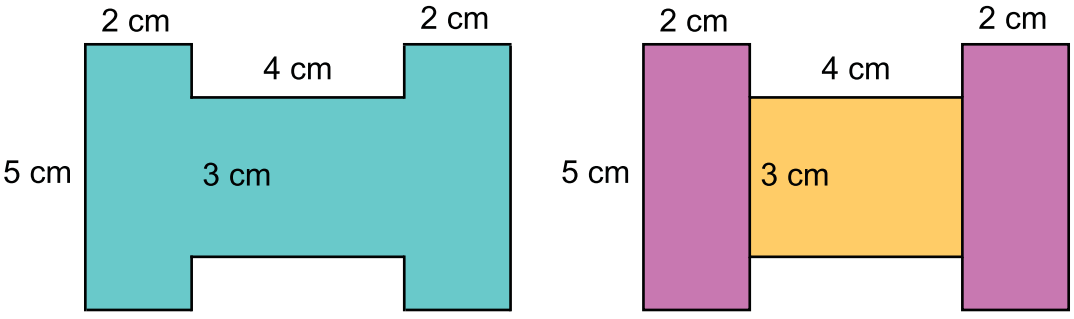
Area of a composite shape

Go online

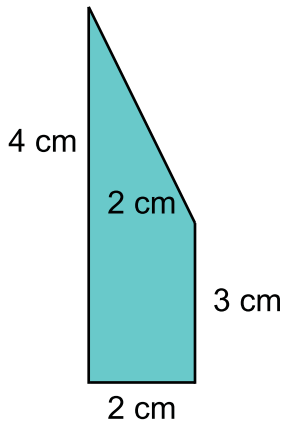
Many shapes can be broken up into rectangles and / or right angled triangles. Look at the shapes below.

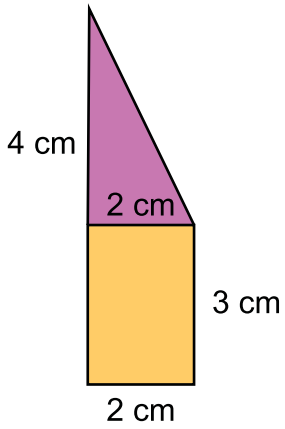


Area of shape = (5 × 1) + (6 × 1) = 11 cm^2

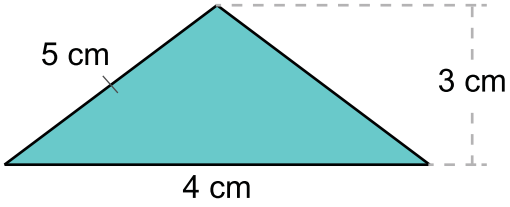


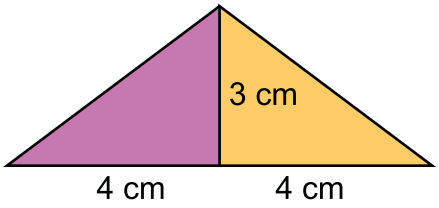
Area of shape = (5 × 2) + (4 × 3) + (5 × 2) = 32 cm^2



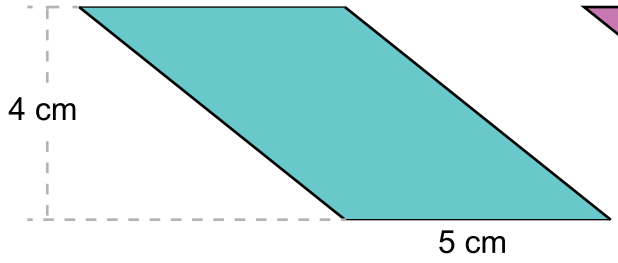


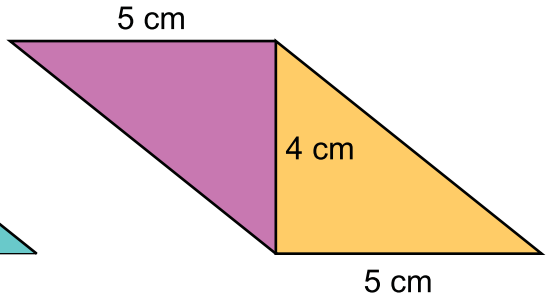
Area of shape = $(2 \times 3) + (\frac{1}{2} \times 4 \times 2) = 10\ cm^2$



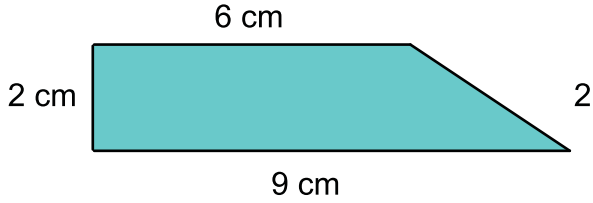


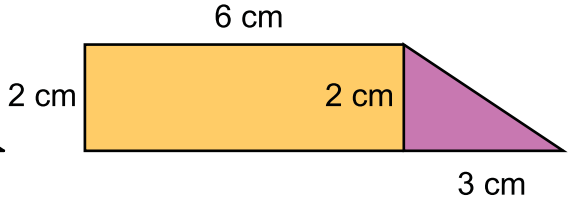
Area of shape = $(\frac{1}{2} \times 3 \times 4) + (\frac{1}{2} \times 3 \times 4) = 12\ cm^2$





Area of shape = $(\frac{1}{2} \times 5 \times 4) + (\frac{1}{2} \times 4 \times 5) = 20\ cm^2$





Area of shape = $(2 \times 6) + (\frac{1}{2} \times 2 \times 3) = 15\ cm^2$

Now let's see how to calculate the area of composite shapes which can be broken down into

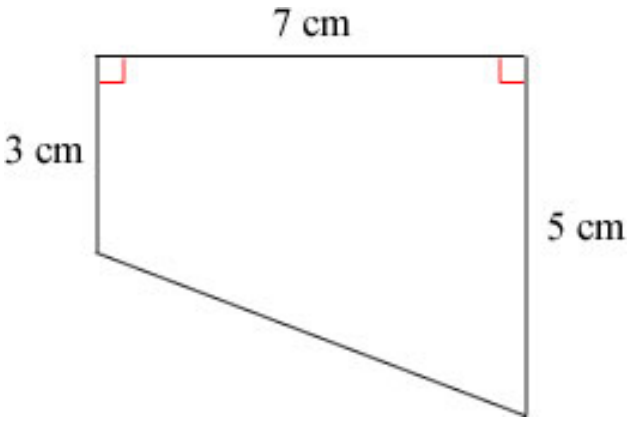
© HERIOT-WATT UNIVERSITY

rectangles and triangles.

Example

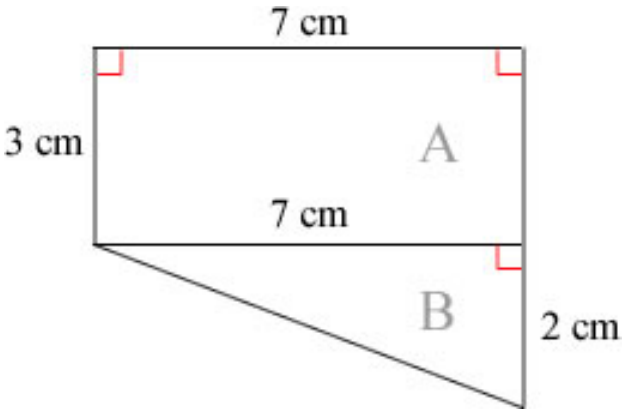
Problem:

Find the area of the shape.



Solution:

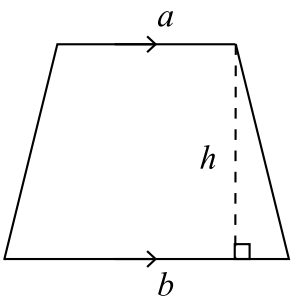
First split the shape up . . .



Area of A (a rectangle) $= 7 \times 3 = 21$ cm^2
Area of B (a right angled triangle) $= {}^1\!/_2 \times 7 \times 2 = 7$ cm^2
Total area $(A + B) = 21 + 7 = 28$ cm^2

Note that the shape above is actually a trapezium.

The diagram below shows a Trapezium.



A trapezium has a pair of parallel sides, a and b and the altitude is labelled h.

There is a formula to calculate the area of a trapezium if you know the values of a, b and h in the diagram.

Key point

The formula for the area of a trapezium is

Area $= {}^{1}/_{2}(a + b)h$

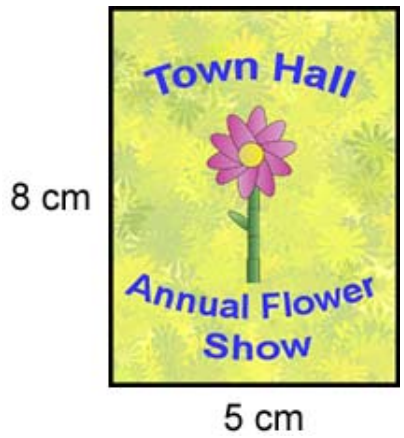
where a and b are the lengths of the parallel sides and h is the altitude.

Composite shapes practice Go online

Q1:

A poster is a rectangle which is 8 cm long by 5 cm wide.

What area does the poster cover? Enter a number without units.



. . . . . .

Q2: What are the units for this area?

a) cm
b) cm^2
c) cm^3

. . . . . .

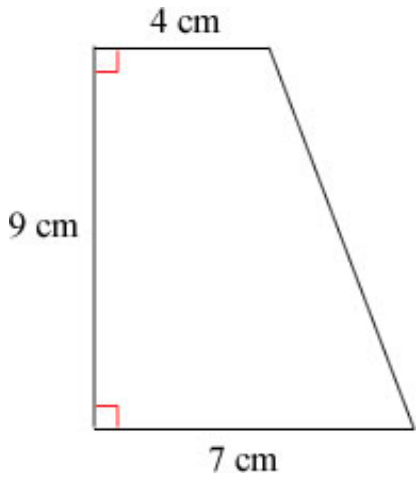
© HERIOT-WATT UNIVERSITY

Q3: Find the area of a right angled triangle which is has a base 4·6 cm long and an altitude of 6·5 cm. Type in a number.

. . . . . .

Q4:

Find the area of the shape.

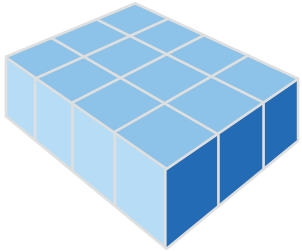


4.1.2 Volume of a cube and a cuboid

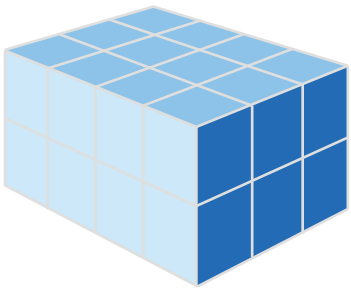
Volume of a Cuboid Go online

This solid has edges 1 cm long. It is a cubic centimetre (1 cm^3).

This layer is made from $3 \times 4 = 12$ cm^3



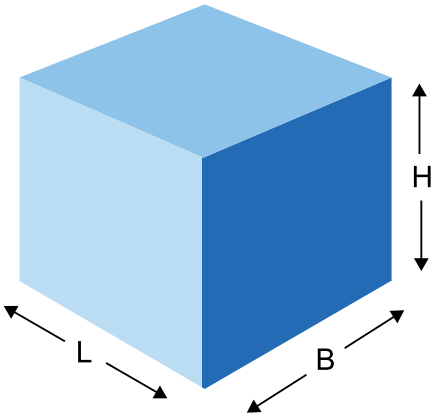
This box hold two layers 2 $\times$ 3 $\times$ 4 = 24 cm^3. It has a volume of 24 cm^3.



The volume of a cuboid can be worked out using the formula:

V = Length $\times$ Breadth $\times$ Height

or in short: $V = L \times B \times H$

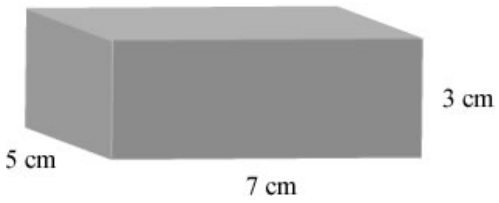


Examples

1. Volume of a cuboid

Problem:

Find the volume of the cuboid shown (measurements are in cm).



Solution:

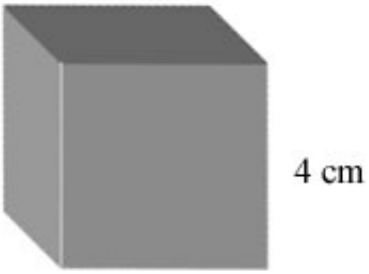
$V = LBH$
$V = 7 \times 5 \times 3$
$V =$105 cm^3

. . . . . .

2. Volume of a cube

Problem:

Find the volume of the cube shown (measurements are in cm).



Solution:

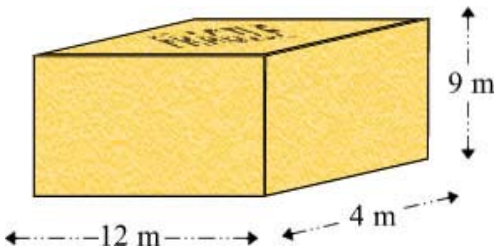
For a cube L = B = H
V = LBH
V = $4 \times 4 \times 4$
V = 64 cm^3

Volume of a cube and cuboid practice Go online

Q5:

Calculate the volume of the box (It's a cuboid).

Enter only a number.



.....

Q6:

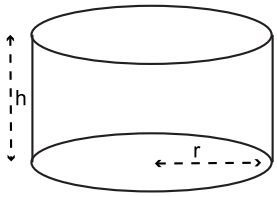
Calculate the volume of the dice (It's a cube).

Enter only a number.

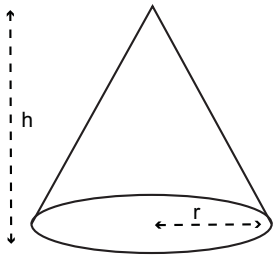
© HERIOT-WATT UNIVERSITY

4.2 Calculate the volume of a solid

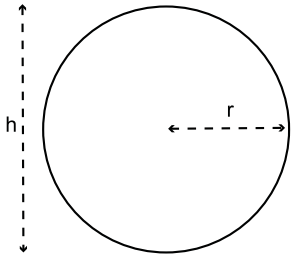
Volume of standard solids Go online



Volume of a cylinder = $\pi r^2 h$



Volume of cone = $^1/_3 \pi r^2 h$
... one third of the volume of a cylinder with the same base and height.



Volume of a cylinder = $\pi r^2 h$

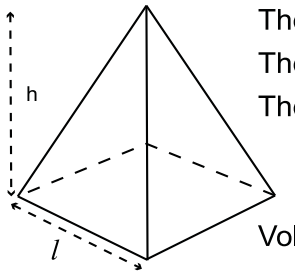
Volume of cone = $^1/_3 \pi r^2 h$

Two cones fit into one sphere of the same height and radius so,

Volume of sphere = $^2/_3 \pi r^2 h$

... two times of the volume of a cone with the same base and height.
... two thirds of the volume of a cylinder with the same base and height.
... the height of a sphere is equal to its diameter, so h = 2r.

Volume of sphere = $^4/_3 \pi r^3$



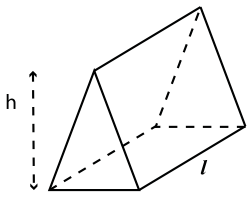
The base of the pyramid is a square.

The area of a square = l^2

The volume of a pyramid

= $^1/_3$ × Area of the base × height

Volume of this Pyramid = $^1/_3 \times l^2 \times h$



The constant cross-section is a triangle.

The area of a triangle = $^1/_2$ the base × the height

The volume of a prism

= area of the constant cross-section × length

Volume of Prism = $^1/_2$ bh × l

Here is a summary of the formulae for the volumes of the solids we have met.

Key point

Volume of a Cuboid = lbh

Volume of a Cylinder = $\pi r^2 h$

Volume of a Cone = $\frac{1}{3}\pi r^2 h$

Volume of a pyramid = $\frac{1}{3} \times Area\ of\ the\ base \times height$

Volume of a Sphere = $\frac{4}{3}\pi r^3$

Volume of a prism = area of the constant cross-section $\times$ length

Remember:

Only the formulae for Cone, Pyramid and Sphere are given on the National 5 formula sheet. The rest *must* be remembered.

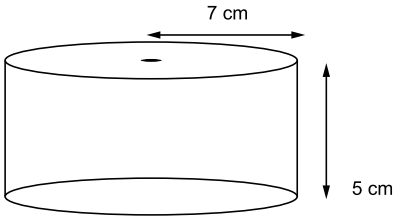
Examples

1.

Problem:

A cylinder has a radius of 7 cm and a height of 5 cm.

Calculate its volume correct to *1 decimal place*.



Solution:

Write your formula... $V = \pi r^2 h$

Substitute... $V = \pi \times 7^2 \times 5$

Evaluate... $V = 769 \cdot 6902001$

Round... $V = 769 \cdot 7\ cm^3$ (to 1 d.p.)

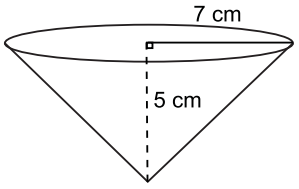
. . . . . .

2.

Problem:

A cone has a radius of 7 cm and a height of 5 cm.

Calculate its volume correct to 1 decimal place.



Solution:

Write your formula... $V = {}^1/_3\pi r^2 h$

Substitute... $V = {}^1/_3\pi \times 7^2 \times 5$

Evaluate... $V = 256 \cdot 5634$

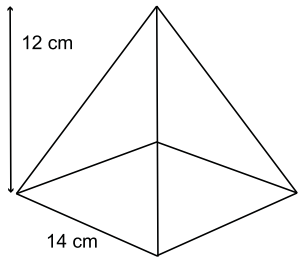
Round... $V = 256 \cdot 6\ cm^3$ (to 1 d.p.)

. . . . . .

3.

Problem:

Calculate the volume of the square based pyramid.



Solution:

Write your formula. . . $V = {}^{1}/_{3} \times area\ of\ base \times height$

Identify the base. . . $V = {}^{1}/_{3} \times l^2 \times h$

Substitute. . . $V = {}^{1}/_{3} \times 14^2 \times 12$

Evaluate. . . $V = 784\ cm^3$

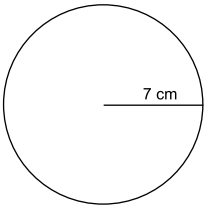
. . . . . .

4.

Problem:

A sphere has a radius of 7 cm.

Calculate its volume correct to 1 decimal place.



Solution:

Write your formula. . . $V = {}^{4}/_{3}\pi r^3$

Substitute. . . $V = {}^{4}/_{3}\pi \times 7^3$

Evaluate. . . $V = 1436 \cdot 75504$

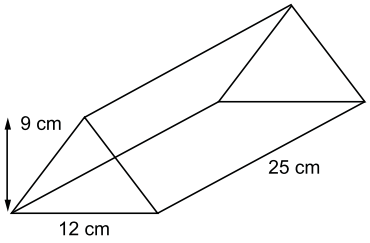
Round. . . $V = 1436 \cdot 8\ cm^3$ (to 1 d.p.)

. . . . . .

5.

Problem:

Calculate the volume of the triangular prism.



© HERIOT-WATT UNIVERSITY

Solution:

Write your formula... $V = area\ of\ constant\ cross-section \times length$

Identify the constant cross-section... $V = {}^1/_2 \times b \times h \times l$

Substitute... $V = {}^1/_2 \times 12 \times 9 \times 25$

Evaluate... $V = 1350\ cm^3$

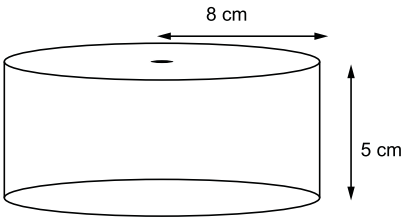
Calculating the volume of standard solids practice

Go online

Q7:

A cylinder has a radius of 8 cm and a height of 5 cm.

Calculate its volume correct to *1 decimal place*.

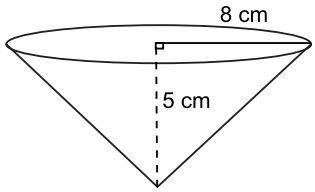


. . . . . .

Q8:

A cone has a radius of 8 cm and a height of 5 cm.

Calculate its volume correct to *1 decimal place*.

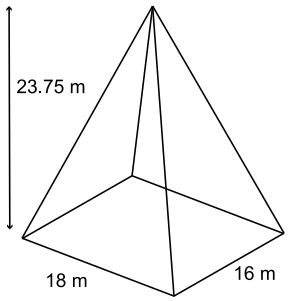


. . . . . .

Q9:

A rectangular pyramid has a length of 16 m, a breadth of 18 m and a height of 23.75 m.

Calculate its volume.

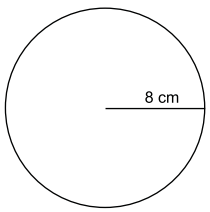


. . . . . .

Q10:

A sphere has a radius of 8 cm.

Calculate its volume correct to *1 decimal place*.

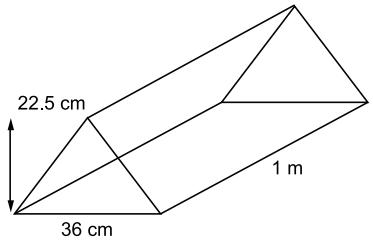


. . . . . .

Q11:

A triangular prism has a length of 1 m, a triangle base of 36 cm and a height of 22·5 cm.

Calculate its volume.



Volume of standard solids exercise

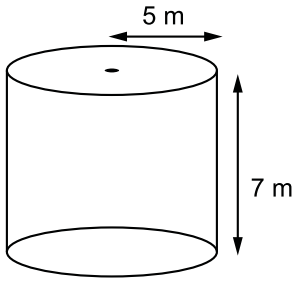
Go online

Q12:

The formula for the volume of a cylinder is
$V = \pi r^2 h$
where r is the radius and h is the height of the cylinder.

Find the volume of a cylinder with a radius of 5 m and a height of 7 m.

What is the volume of the cylinder in m^3? Give your answer correct to **1 decimal place**.



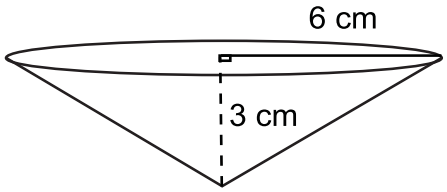
. . . . . .

Q13:

The formula for the volume of a cone is
$V = {}^1/_3 \pi r^2 h$
where r is the radius and h is the height of the cone.

Find the volume of a cone with a radius of 6cm and a height of 3 cm.

What is the volume of the cone, in cm^3, correct to **2 decimal places**?



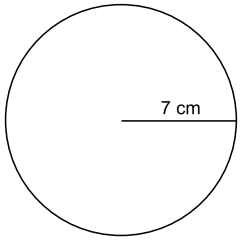
. . . . . .

Q14:

The formula for the volume of a sphere is
$V = {}^4/_3 \pi r^3$
where r is the radius of the sphere.

A sphere has a radius of 7 cm.

What is the volume of the sphere, in cm^3, correct to the **nearest whole number**?



. . . . . .

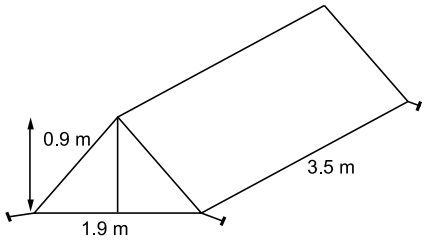
© HERIOT-WATT UNIVERSITY

Q15:

The tent is in the shape of a triangular prism. The formula for the volume of a prism is
$V = area\ of\ the\ constant\ cross$ section $\times$ $length$

The tent has a height of 0·9 m, base of 1·9 m and length of 3·5 m.

Calculate the volume of triangular prism, correct to **3 significant figures**.



. . . . . .

Q16:

The picture shows the Great Pyramid of Giza in Egypt. The formula for the volume of a pyramid is
$V = {}^{1}/_{3} \times area\ of\ the\ base \times height$

It is a square-based pyramid of length 230·4 m and height 146·4 m.

Calculate the volume of the Great Pyramid of Giza, correct to **3 significant figures**.

4.3 Finding an unknown dimension when you know the volume

By re-arranging the formula for the volume we can find an unknown dimension.

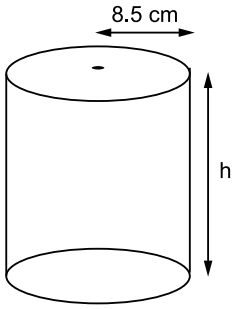
Examples

1.

Problem:

If the volume of the cylinder is 4063 cm^3, calculate its height.

Give your answer to 1 decimal place.



Solution:

The formula for the volume of a cylinder is $V = \pi r^2 h$ where radius $r = 8 \cdot 5\ cm$.
If we substitute the volume and the radius we get,

$$\begin{aligned} 4063 &= \pi \times 8 \cdot 5^2 \times h \qquad \left(\pi \times 8 \cdot 5^2 = 226 \cdot 98\right) \\ 4063 &= 226 \cdot 98 \times h \qquad (rearrange\ the\ equation) \\ \frac{4063}{226 \cdot 98} &= h \\ h &= 17 \cdot 9\ cm \ \ (to\ 1\ d.p.) \end{aligned}$$

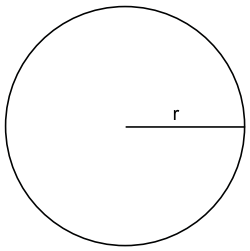
. . . . . .

2.

Problem:

If the volume of the sphere is 14·1 m^3, calculate its radius.

Give your answer to 1 decimal place.



Solution:

The formula for the volume of a sphere is $V = {}^4/_3 \pi r^3$.
If we substitute the volume we get

$$\begin{aligned} 14 \cdot 1 &= \frac{4}{3} \times \pi \times r^3 \qquad \left(\frac{4}{3} \times \pi = 4 \cdot 189\right) \\ 14 \cdot 1 &= 4 \cdot 189 \times r^3 \qquad (re-arrange\ the\ equation) \\ \frac{14 \cdot 1}{4 \cdot 189} &= r^3 \qquad (find\ the\ cube\ root) \\ \sqrt[3]{\frac{14 \cdot 1}{4 \cdot 189}} &= r \\ r &= 1 \cdot 5\ m\ \ (to\ 1\ d.p.) \end{aligned}$$

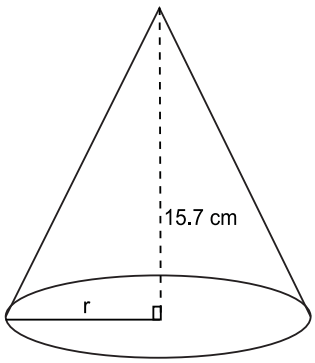
. . . . . .

3.

Problem:

If the volume of the cone is 924·8 cm^3, calculate its radius.

Give your answer to 1 decimal place.



Solution:

The formula for the volume of a cone is $V = {}^1/_3\pi r^2 h$ with height $h = 15 \cdot 7\ cm$.
If we substitute the volume and the height we get,

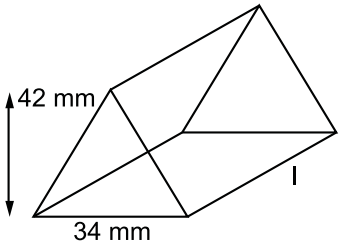
$$\begin{aligned}
924 \cdot 8 &= \frac{1}{3} \times \pi \times r^2 \times 15 \cdot 7 \quad \left(\frac{1}{3} \times \pi \times 15 \cdot 7 = 16 \cdot 441\right) \\
924 \cdot 8 &= 16 \cdot 441 \times r^2 \qquad (re-arrange\ the\ equation) \\
\frac{924 \cdot 8}{16 \cdot 441} &= r^2 \qquad (find\ the\ square\ root) \\
\sqrt{\frac{924 \cdot 8}{16 \cdot 441}} &= r \\
r &= 7 \cdot 5\ cm\ \ (to\ 1\ d.p.)
\end{aligned}$$

. . . . . .

4.

Problem:

If the volume of the prism is 39270 mm^3, calculate its length.



Solution:

The formula for the volume of a triangular prism is $V = area\ of\ the\ constant\ cross\ section \times length = {}^1/_2 \times b \times h \times l$ with height $h = 42\ mm$ and base $b = 34\ mm$.
If we substitute the volume, base and height of the triangle we get

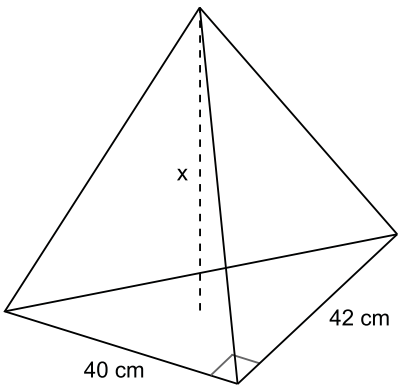
$$\begin{aligned}
39270 &= \frac{1}{2} \times 34 \times 42 \times l \quad \left(\frac{1}{2} \times 34 \times 42 = 714\right) \\
39270 &= 714 \times l \qquad (re-arrange\ the\ equation) \\
\frac{39270}{714} &= l \\
l &= 55\ mm
\end{aligned}$$

. . . . . .

5.

Problem:

If the volume of the pyramid is 14560 cm^3, calculate its height.



Solution:

The formula for the volume of a triangular pyramid is $V = {}^1/_3 \times area\ of\ base \times height$
The base of the pyramid is a triangle and the formula for the area of a triangle is $A = {}^1/_2\, bh$.
If we substitute the volume, base and height of the triangle we get

$$\begin{aligned} 14560 &= \frac{1}{3} \times \frac{1}{2} \times 40 \times 42 \times h \qquad \left(\frac{1}{3} \times \frac{1}{2} \times 40 \times 42 = 280\right) \\ 14560 &= 280 \times h \qquad (re-arrange\ the\ equation) \\ \frac{14560}{280} &= h \\ h &= 52\ cm \end{aligned}$$

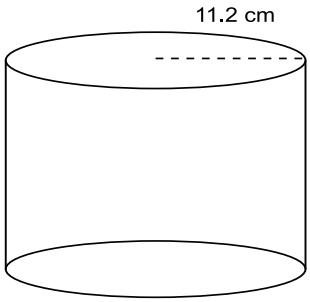
Finding an unknown dimension given the volume practice

Go online

Q17:

Calculate the height of the cylinder if its volume is 7251 cm^3 and its radius is 11·2 cm.

Give your answer to 1 decimal place.

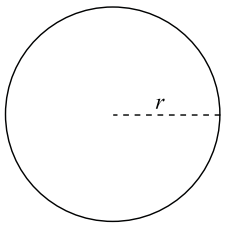


. . . . . .

Q18:

Calculate the radius of the sphere if its volume is 7·2 m^3.

Give your answer to 1 decimal place.

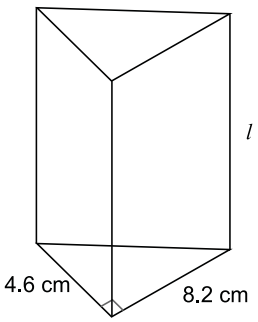


. . . . . .

Q19:

Calculate the length of the triangular prism if its volume is 358 cm^3.

Give your answer to 1 decimal place.



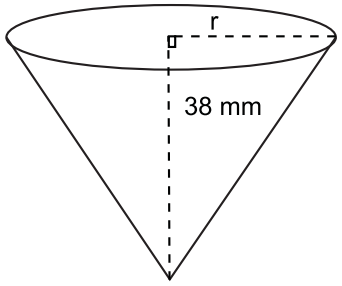
. . . . . .

© HERIOT-WATT UNIVERSITY

Q20:

Calculate the radius of the cone if its volume is 8125 mm^3.

Give your answer to 1 decimal place.

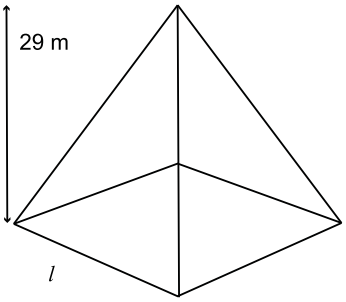


. . . . . .

Q21:

Calculate the length of the base of the square-based pyramid if its volume is 5203 m^3.

Give your answer to 1 decimal place.



Finding an unknown dimension given the volume exercise

Go online

Q22: Finding the radius of a cylinder.

The volume of the tin of beans is 507·5 cm^3.

Given that the height of the cylinder is 11·7 cm, calculate the radius of the tin giving your answer correct to 1 decimal place.

. . . . . .

Q23: Finding the height of a cone.

The volume of ice-cream which this cone could contain is 81·8 ml.

If the cone has a radius of 2·5 cm, calculate the height of the cone.

Give your answer correct to 3 significant figures. (Remember: 1 ml = 1 cm^3)

. . . . . .

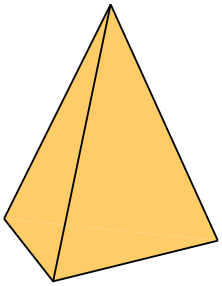
Q24: Finding the radius of a sphere.

Planet Earth is a sphere. The volume of our planet is $1 \cdot 08321 \times 10^{12}\ km^3$.

Calculate the radius of the sphere correct to the nearest km.

. . . . . .

Q25: Finding the height of a pyramid.



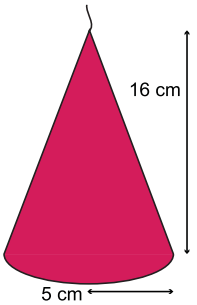
The pyramid has a rectangular base of length 102 m and breadth 84 m.

Calculate the height of the pyramid if its volume is 574000 m^3.

Give your answer correct to 3 significant figures.

. . . . . .

Q26:



A company makes candles in the shape of cones and pyramids.

The cone has a radius of 5 cm and a height of 16 cm.

Calculate the volume of the cone, giving your answer correct to 3 significant figures.

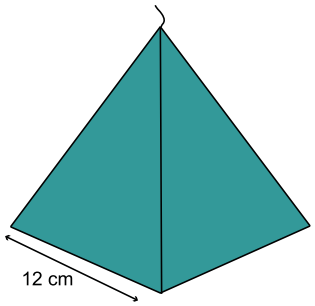
. . . . . .

© HERIOT-WATT UNIVERSITY

Q27:

The pyramid has a square base of side 12 cm and the same volume as the cone in the question above.

Calculate the height of the pyramid, correct to 1 decimal place.



4.4 Volumes of composite solids

Composite solids are shapes which are composed of 2 or more solids.

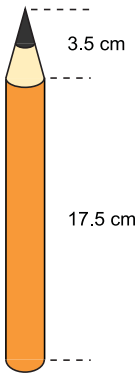
Examples

1.

Problem:

The pencil is 21 cm long and has a radius of 0·5 cm.

Calculate the volume of the pencil, correct to 1 decimal place.



Solution:

This shape is made from a cylinder and a cone and both shapes have the same radius.
We need Volume of a Cylinder $= \pi r^2 h$ and Volume of a Cone $= {}^1/_3 \pi r^2 h$
Volume of a Cylinder $= \pi \times 0 \cdot 5^2 \times 17 \cdot 5 = 13 \cdot 7\ cm^3$
Volume of a Cone $= {}^1/_3 \times \pi \times 0 \cdot 5^2 \times 3 \cdot 5 = 0 \cdot 9\ cm^3$
Total Volume $= 13 \cdot 7 + 0 \cdot 9 = 14 \cdot 6 cm^3$

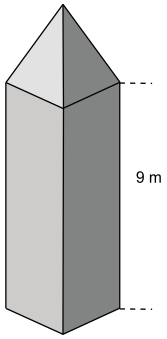
. . . . . .

2.

Problem:

The height of the obelisk is 10·8 m and has a square base of side 2·6 m.

Calculate the volume of the obelisk, correct to 3 significant figures.



Solution:

This shape is made from a cuboid and a square-based pyramid and both have the same length of a side.
We need Volume of a Cuboid $= lbh$ and Volume of a pyramid $= {}^1/_3 \times Area\ of\ the\ base \times height$

The height of the pyramid $= 10 \cdot 8 - 9 = 1 \cdot 8\ m$
Volume of a Cuboid $= 2 \cdot 6 \times 2 \cdot 6 \times 9 = 60 \cdot 84 cm^3$
Volume of a pyramid $= {}^1/_3 \times 2 \cdot 6^2 \times 1 \cdot 8 = 4 \cdot 056\ cm^3$
Total Volume = $60 \cdot 84 + 4 \cdot 056 = 64 \cdot 896$
= $64 \cdot 9\ cm^3$ to 3 significant figures

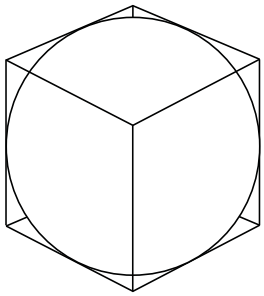
. . . . . .

3.

Problem:

A ball of radius 2·2 cm fits perfectly inside a box.

Calculate the volume of space in the box.



Solution:

This shape shows a cube with a sphere inside. The side of the cube is the same as the diameter of the sphere.
We need Volume of a Cube $= l^3$ and Volume of a Sphere $= {}^4/_3 \pi r^3$

The length of a side on the cube $= 2 \times 2 \cdot 2 = 4 \cdot 4$
Volume of Cube $= 4 \cdot 4^3 = 85 \cdot 184 cm^3$
Volume of Sphere $= {}^4/_3 \times \pi \times 2 \cdot 2^3 = 44 \cdot 602\ cm^3$
Volume of space $= 85 \cdot 184 - 44 \cdot 602 = 40 \cdot 582$
$= 40 \cdot 6\ cm^3$

© HERIOT-WATT UNIVERSITY

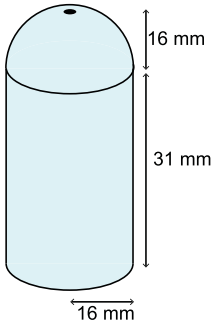
Volumes of composite solids exercise

Go online

Q28:

The salt shaker is 47mm tall and has a radius of 16 mm.

Calculate the volume of the salt shaker, correct to 1 decimal place. (Hint: the radius of the cylinder and hemisphere are the same.)



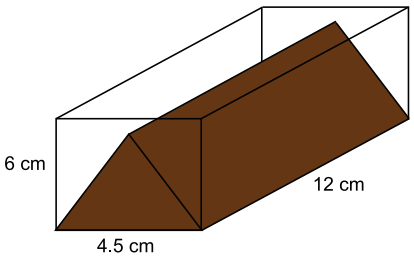
. . . . . .

Q29:

A triangular prism of chocolate is placed in a cuboid.

The height of the triangular prism is 5·2 cm and the length of its base is 4·5 cm.

Calculate the volume of space left in the box correct to 3 significant figures.

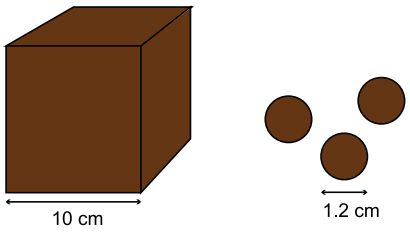


. . . . . .

Q30:

A 10 cm cube of chocolate is melted down.

How many complete spheres with diameter 1·2 cm can be formed from the chocolate?
(Hint: Do not round your answer for the volume of a sphere of chocolate.)



. . . . . .

Q31:

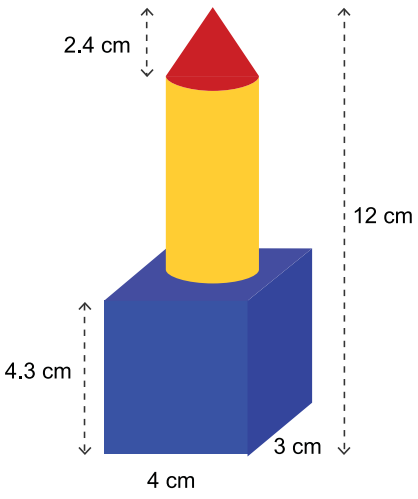
A Child's wooden toy is made from a cuboid, cylinder and cone.

The cone fits perfectly on top of the cylinder.

The radius of the cone is 1·1 cm.

Calculate the volume of wood needed to make the toy.

Give your answers correct to 2 decimal places.



© HERIOT-WATT UNIVERSITY

4.5 Learning points

- Volume of a cuboid = lbh
- Volume of a cylinder = $\pi r^2 h$
- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Volume of a pyramid = $\frac{1}{3} \times Area\ of\ the\ base \times height$
- Volume of a sphere = $\frac{4}{3}\pi r^3$
- Volume of a prism = area of the constant cross-section $\times$ length
- You may also need to remember these formulae:
  - Area of a triangle = $\frac{1}{2}bh$
  - Area of a square = l^2
  - Area of a rectangle = lb
- To find an unknown dimension on a solid shape when you know its volume:
  - put the values you know into the formula including the volume;
  - rearrange or change the subject of the formula.

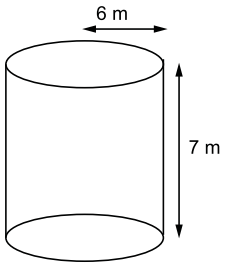
© HERIOT-WATT UNIVERSITY

4.6 End of topic test

End of topic 4 test Go online

Q32:

Volume of a cylinder



Find the volume of a cylinder with a radius of 6 m and a height of 7 m.

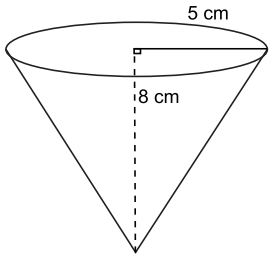
What is the volume of the cylinder in m^3? Give your answer correct to **1 decimal place**.

.....

Q33:

Volume of a cone

The formula for the volume of a cone is
$V = {}^1/_3\pi r^2 h$
where r is the radius and h is the height of the cone.



Find the volume of a cone with a radius of 5 cm and a height of 8 cm.

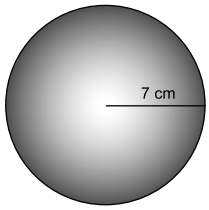
What is the volume of the cone, in cm^3, correct to **2 decimal places**?

.....

Q34:

Volume of a sphere

The formula for the volume of a sphere is
$V = {}^4/_3\pi r^3$
where r is the radius of the sphere.



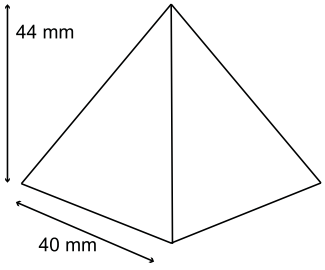
A sphere has a radius of 7 cm.

What is the volume of the sphere, in cm^3, correct to the **nearest whole number**?

.....

Q35:

Volume of a pyramid



A Paperweight in the shape of a square-based pyramid has length 40 mm and height 44 mm.

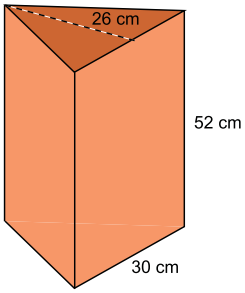
Calculate the volume of the paperweight.

Give your answer correct to 3 significant figures.

. . . . . .

Q36:

Volume of a prism



A wastepaper bin is in the shape of a triangular prism.

It is 52 cm tall and has a triangular base of side 30 cm and height 26 cm.

Calculate the volume of the wastepaper bin.

. . . . . .

Q37:

Finding the height of a solid

The glass beaker holds 1400 ml. If the radius is 5·2 cm, calculate the height of the beaker.

Give your answer correct to 1 decimal place. (Hint: 1 ml = 1 cm^3)

. . . . . .

Q38:

Finding the radius of a solid

A globe of the world has a volume of 65500 cm^3, calculate the radius of the globe correct to the nearest centimetre.

. . . . . .

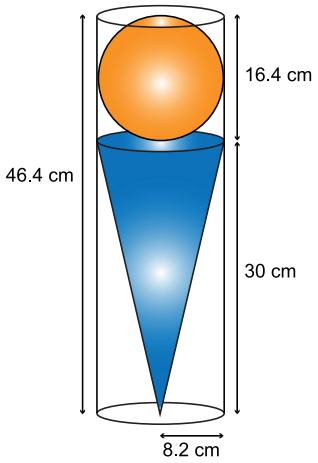
© HERIOT-WATT UNIVERSITY

Q39:

Volume of a composite solid

A cone and a sphere are placed inside a cylinder.

Calculate the volume of space not used in the cylinder correct to 3 significant figures.



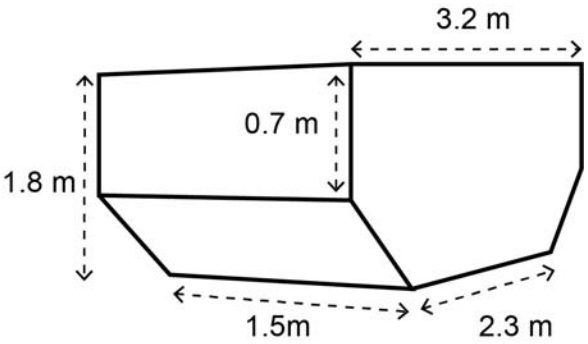
a) $V = {}^{1}/_{3}\pi r^{2}$h

b) $V = {}^{4}/_{3}\pi r^{3}$

. . . . . .

Q40:

An airline container is in the shape of a prism.



Calculate the volume of the airline container correct to 2 significant figures.

Answers to questions and activities

Topic 4: Volumes of solids

Composite shapes practice (page 8)

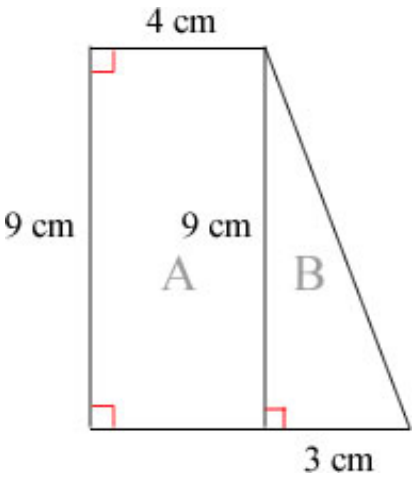
Q1: 40 cm^2

Q2: b) cm^2

Q3: 14.95 cm^2

Q4:

You must first split up the shape.



Area of $A = lb = 9 \times 4 = 36$ cm^2

Area of $B = {}^1/_2 \times base \times altitude = {}^1/_2 \times 3 \times 9 = 13{\cdot}5$ cm^2

Total area $(A + B) = 36 + 13 \cdot 5 = 49 \cdot 5$ cm^2

Volume of a cube and cuboid practice (page 11)

Q5: 432 m^3

Q6: 125 cm^3

Calculating the volume of standard solids practice (page 15)

Q7: 1005.3 cm^3

Q8: 335.1 cm^3

Q9: 2280 m^3

Q10: 2144.7 cm^3

Q11: 40500 cm^3

© HERIOT-WATT UNIVERSITY

Volume of standard solids exercise (page 16)

Q12: 549·8 m^3

Q13: 113·10 cm^3

Q14: 1437 cm^3

Q15:

Constant cross section = triangle

Area of a triangle $= {}^1/_2 bh = 0 \cdot 855\ m^3$

2·99 m^3

Q16:

Base of the pyramid = square

Area of a square $= l^2 = 53084 \cdot 16\ m^3$

2590000 m^3

Finding an unknown dimension given the volume practice (page 20)

Q17: 18.4 cm

Q18: 1.2 m

Q19: 19.0 cm

Q20: 14.3 mm

Q21: 23.2 m

Finding an unknown dimension given the volume exercise (page 21)

Q22:

Volume of a cylinder $= \pi \times r^2 \times h$

$\pi \times h = \pi \times 11 \cdot 7 = 36 \cdot 75663404700\ldots$

3·7 cm

Q23:

Volume of a cone $= {}^1/_3 \times \pi \times r^2 h$

${}^1/_3 \times \pi \times r^2 = {}^1/_3 \times \pi \times 2 \cdot 5^2 = 6.544984694\ldots$

12·5 cm

Q24:

Volume of a sphere $= {}^4/_3 \times \pi \times r^3$

${}^4/_3 \times \pi = 4.188790204\ldots$

6371 km

© HERIOT-WATT UNIVERSITY

Q25:

Volume of a pyramid $= {}^1/_3 \times area\ of\ the\ base \times h$

${}^1/_3 \times area\ of\ the\ base = {}^1/_3 \times 102 \times 84 = 2856$

201 m

Q26: 419 cm^3

Q27: 8·7 cm

Volumes of composite solids exercise (page 25)

Q28:

Volume of half of a sphere = 8578·6 mm^3
Volume of cylinder = 24931·7 mm^3
Total = 33510·3 mm^3

Q29:

Volume of the cuboid = 324 cm^3
Volume of the triangular prism = 140·4 cm^3
Volume of space = 183·6 cm^3
3 significant figures = 184 cm^3

Q30:

Volume of cube = 1000 cm^3
Volume of 1 sphere = 0·90477868... cm^3
Number of complete spheres = 1105

Q31:

Steps:

- Volume of the Cuboid $\rightarrow 4 \cdot 3 \times 4 \times 3 = 51 \cdot 6\ cm^3$
- Volume of the Cone $\rightarrow {}^1/_3 \times \pi \times 1 \cdot 1^2 \times 2 \cdot 4 = 3 \cdot 04\ cm^3$
- Radius of the cylinder $\rightarrow 1 \cdot 1\ cm$
- Height of the cylinder $\rightarrow 12 - (2 \cdot 4 + 4 \cdot 3) = 5 \cdot 3\ cm$
- Volume of the cylinder $\rightarrow 20 \cdot 15\ cm^3$

Answer: Total volume of wood = 74·8 cm^3

End of topic 4 test (page 27)

Q32: 791·7 m^3

Q33: 209·44 cm^3

Q34: 1437 cm^3

Q35: 23500 mm^3

© HERIOT-WATT UNIVERSITY

Q36: 20280 cm^3

Q37: 16·5 cm^3

Q38: 25 cm

Q39:

Steps:

- Volume of the cylinder correct to 4 significant figures = 9802 cm^3
- Volume of the cone correct to 4 significant figures = 2112 cm^3
- Volume of the sphere correct to 4 significant figures= 2310 cm^3

Answer: Volume of space not used in the cylinder = 5380 cm^3

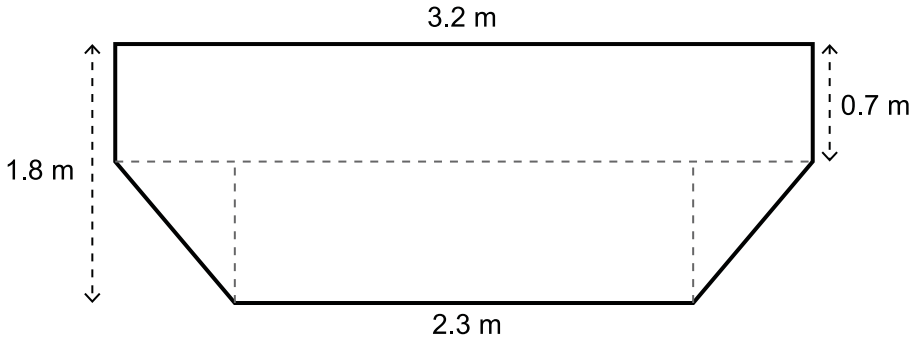
Q40:

Hints:

- Split the area of the front of the container up into smaller familiar shapes.

Steps:

a)



b) Area of the large rectangle correct to ? significant figures = 2·24 m^2

c) Area of the small rectangle correct to ? significant figures = 2·53 m^2

d) Area of the two triangles correct to ? significant figures = 0·495 m^2

e) Total area of constant cross section = 5·265 m^2

Answer: Volume of airline container = 7·9 m^3

© HERIOT-WATT UNIVERSITY