SCHOLAR Study Guide

National 5 Mathematics

Course Materials Topic 8: Expanding brackets

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Topic 8

Expanding brackets

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Learning objective

By the end of this topic, you should be able to:

- multiply out single brackets;
- multiply out double brackets.

8.1 Looking back at National 4: Expressions involving brackets

Expressions Involving Brackets

Go online



First let us look at how to multiply an expression involving brackets.

4 threes $=4\times3=3+3+3+3=12$: Multiplying by 4 is the same as adding 3 four times.

It is the same for letters...

4 d's = $4 \times d = d + d + d + d = 4d$: Multiplying by 4 is the same as adding the letter four times.

Let's look at what happens with expressions...

$$4(d + 3) = 4 \times (d + 3) = (d + 3) + (d + 3) + (d + 3) + (d + 3)$$

Add the expression four times...

$$4(d+3) = 4 \times (d+3) = d+3+d+3+d+3+d+3$$

Gather like terms together...

$$4(d+3) = 4 \times (d+3) = d+d+d+d+3+3+3+3$$

$$4(d + 3) = 4 d$$
's $+ 4$ threes $= 4d + 12$

$$4(d + 3) = 4d + 12$$

Now look at how to simplify an expression involving brackets.

Before you can simplify, get rid of any brackets...

$$3(x+5) + 2 = 3x + 3 \times 5 + 2$$
$$= 3x + 15 + 2$$
$$a = 3x + 17$$

Another example...

$$4(x+3) + 2x = 4x + 4 \times 3 + 2x$$
$$= 4x + 12 + 2x$$
$$= 6x + 12$$

Examples

1.

Problem:

Multiply out the brackets 5(x-2).

Solution:

$$5(x-2) = 5x - 5 \times 2$$
$$= 5x - 10$$

4

2.

Problem:

Simplify 5(x + 1) - 3x.

Solution:

$$5(x+1) - 3x = 5x + 5 \times 1 - 3x$$
$$= 5x + 5 - 3x$$
$$= 2x + 5$$

Key point

Remember:

- each term inside the bracket is multiplied by the term outside the bracket;
- simplify by collecting like terms after you have expanded the bracket.

Q1:

Multiply out the brackets 6(x + 4).

......

Q2:

Simplify 7(x + 1) - 2x.

8.2 Expand single bracket expressions

Multiplying by a number

Go online



The following examples shows you how to multiply algebraic expressions by a number.

6 fours =
$$6 \times 4 = 4 + 4 + 4 + 4 + 4 + 4$$

$$6g = 6 \times g = g + g + g + g + g + g$$

$$6(g + 4) = 6g + 6 \times 6$$

= 6g + 24

$$5(d + b) = 5 \times (d + b)$$

$$= (d + b) + (d + b) + (d + b) + (d + b) + (d + b)$$

$$= d + d + d + d + d + b + b + b + b + b$$

$$5(d + b) = 5d + 5b$$

Examples

1.

Problem:

Expand 3(x + 4).

Solution:

$$3(x + 4) = (x + 4) + (x + 4) + (x + 4)$$

$$= x + 4 + x + 4 + x + 4$$

$$= 3x + 3 \times 4$$

$$= 3x + 12$$

2.

Problem:

Expand -2(x-3).

Solution:

$$\begin{array}{rcl}
-2(x - 3) & = & -2 \times x + -2 \times -3 \\
 & = & -2x + 6
\end{array}$$

Multiplying by a number practice

Go online



Q3: Expand 2(x - 1).

.....

Q4: Expand 4(5x - 2).

Multiplying by a letter

Go online



The following example shows you how to multiply algebraic expressions by a letter.

$$a(b+f) = a \times (b+f)$$

$$= a \times b + a \times f$$

$$a(b+f) = ab + af$$

$$b(c+d) = b \times (c+d)$$

$$= b \times c + b \times d$$

Examples

1.

Problem:

Expand x(3x + 2).

Solution:

$$x(3x + 2) = x \times 3x + x \times 2$$
$$= 3x^2 + 2x$$

2.

Problem:

Expand x(3x + 4y).

Solution:

$$x(3x + 4y) = x \times 3x + x \times 4y$$
$$= 3x^2 + 4xy$$

Multiplying by a letter practice

Go online



Q5: Expand y (3y + 1)

.....

Q6: Expand y(3y + 5k).

Multiplying by a number and a letter

Go online



The following example shows you how to multiply algebraic expressions by a number and a letter.

$$3a(a + 4) = 3a \times (a + 4)$$

= $3a \times a + 3a \times 4$
 $3a(a + 4) = 3a^{2} + 12a$

Examples

1.

Problem:

Expand 5g(3g-2)

Solution:

$$5g(3g-2) = 5g \times 3g + 5g \times (-2)$$
$$= 15g^2 - 10g$$

2.

Problem:

Expand $2h^2(4h + 2k)$

Solution:

$$2h^{2}(4h + 2k) = 2h^{2} \times 4h + 2h^{2} \times 2k$$
$$= 8h^{3} + 4h^{2}k \quad or \quad 8h^{3} + 4kh^{2}$$

Multiplying by a number and a letter practice

Go online



Q7: Expand 7w(2w + 3)

Q8: Expand $2m(3m^2 - 9n)$

Single bracket expressions





The following example shows you how to multiply single bracket algebraic expressions.

$$3(b+2)+4(b+1) = 3 \times b + 3 \times 2 + 4 \times b + 4 \times 1$$

= $3b + 6 + 4b + 4$
 $3(b+2)+4(b+1) = 7b + 10$

The following example shows you how to multiply negative single bracket algebraic expressions.

$$4(3p-1) - 2(p-5) = 4 \times 3p + 4 \times (-1) + (-2) \times p + (-2) \times (-5)$$

= $12p - 4 - 2p + 10$
 $4(3p-1) - 2(p-5) = 10p + 6$

Examples

1.

Problem:

Expand and simplify 3(3t+5) + 4(2t-1)

Solution:

$$3(3t+5) + 4(2t-1) = 3 \times 3t + 3 \times 5 + 4 \times 2t + 4 \times (-1)$$
$$= 9t + 15 + 8t + (-4)$$
$$= 17t + 11$$

2.

Problem:

Expand and simplify r(5r + 4) + 3(r - 2)

Solution:

$$r(5r + 4) + 3(r - 2) = r \times 5r + r \times 4 + 3 \times r + 3 \times (-2)$$

= $5r^2 + 4r + 3r - 6$
= $5r^2 + 7r - 6$

3.

Problem:

Expand and simplify 5(3d - 2) - 3(d + 6)

Solution:

$$5(3d - 2) - 3(d + 6) = 5 \times 3d + 5 \times (-2) + (-3) \times d + (-3) \times 6$$
$$= 15d - 10 - 3d - 18$$
$$= 12d - 28$$

Expanding single brackets practice

Go online



Q9: Expand and simplify 10(z + 3) + 2(z - 12)

.....

Q10: Expand and simplify 3(f + 4) - (f - 3)

.....

Q11: Expand and simplify e(2e + 8) - 2(e - 5)

Expanding single brackets exercise

Go online



These questions are for practice only.

Q12: Expand 2(x - 1)

.....

Q13: Expand 5(-4x + 1)

......

Q14: Expand x(2x + 4)...
Q15: Expand x(-2x + y)...
Q16: Expand 2x(5x - 3)...
Q17: Expand 3y(4y + 2z)...
Q18: Expand 7y(2x + 3y - z)...
Q19: Expand and simplify 3(x - 5) + 4(x + 3)Q20: Expand and simplify y(2y + 3) + 2(3y - 1)...
Q21: Expand and simplify 6(3z + 4) - 5(z - 2)

8.3 Expand double bracket expressions

Multiplying expressions

Go online



The following example shows how to multiply algebraic expressions by an expression.

$$(ax + b) \times (cx + d) = ax(cx + d) + b(cx + d)$$

= $acx^2 + adx + bcx bd$

$$(x + 5)(x + 2) = x(x + 2) + 5(x + 2)$$

= $x^2 + 2x + 5x + 10$
 $(x + 5)(x + 2) = x^2 + 7x + 10$

The method above has three lines of working but there are lots of other methods for multiplying out double brackets and some of these require very little working.

Top tip

The method called **FOIL** tells you which terms to multiply together:

- F First terms
- O Outer terms
- I Inner terms
- L Last terms

The Rainbow method requires you to link the terms you multiply together with a loop.

Try the activity below and use the method you prefer.

Using the FOIL/rainbow method







Examples

1.

Problem:

Expand and simplify (x+4)(x+1)

Solution:

$$(x+4)(x+1) = x(x+1) + 4(x+1)$$
$$= x^2 + x + 4x + 4$$
$$= x^2 + 5x + 4$$

2.

Problem:

Expand and simplify (x+4)(x-1)

Solution:

$$(x+4)(x-1) = x(x-1) + 4(x-1)$$

$$= x^2 - x + 4x - 4$$

$$= x^2 + 3x - 4$$

3.

Problem:

Expand and simplify (x-4)(x-2)

Solution:

$$(x-4)(x-2) = x(x-2) - 4(x-2)$$

$$= x^2 - 2x - 4x + 8$$

$$= x^2 - 6x + 8$$

4.

Problem:

Expand and simplify (2x+3)(x-5)

Solution:

$$(2x+3)(x-5) = 2x(x-5) + 3(x-5)$$
$$= 2x^{2} - 10x + 3x - 15$$
$$= 2x^{2} - 7x - 15$$

Multiplying double bracket expressions practice





Q22: Expand and simplify (x+6)(x+4)

.....

Q23: Expand and simplify (a+3)(a-4)

.....

Q24: Expand and simplify (g-2)(g+3)

.....

Q25: Expand and simplify (k-1)(k-5)

.....

Q26: Expand (2x+1)(x+3)

.....

Q27: Expand and simplify (3h-2)(h+1)

.....

Q28: Expand and simplify (2m-3)(2m-1)

.....

Q29: Expand and simplify (4t+1)(2t-3)

We are now going to look at a special case when expanding brackets.

A special case: The difference of two squares

Go online



The following example shows the difference of two squares.

$$(p + m)(p - m) = p(p - m) + m(p - m)$$

= $p^2 - mp + mp - m^2$
= $p^2 - m^2$

Examples

1.

Problem:

Expand and simplify (x+5)(x-5)

Solution:

$$(x+5)(x-5) = x(x-5) + 5(x-5)$$

= $x^2 - 5x + 5x - 25$
= $x^2 - 25$

2.

Problem:

Expand and simplify (2a + 3)(2a - 3)

Solution:

$$(2a+3)(2a-3) = 2a(2a-3) + 3(2a-3)$$
$$= 4a^2 - 6a + 6a - 9$$
$$= 4a^2 - 9$$

Difference of two squares practice

Go online



Q30: Expand and simplify (b+4)(b-4)

......

Q31: Expand and simplify (2g+7)(2g-7)

Expanding more complex double brackets

We are now going to look at another example of a special case when expanding brackets.

Multiplying more complex expressions





The following example shows how to multiply more complex algebraic expressions.

$$(x+2)(x^2+3x-1) = x(x^2+3x-1) + 2(x^2+3x-1)$$

$$Multiply the second bracket by x then by +2$$

$$= x^3 + 3x^2 - x + 2x^2 + 6x - 2$$

$$Multiply out the single brackets$$

$$= x^3 + 5x^2 - 5x - 2$$

$$Collect like terms$$

Top tip

You should show <u>all</u> working for this type of question if you hope to gain full marks.

Examples

1.

Problem:

Expand and simplify $(y+3)(y^2-2y+4)$

Solution:

$$(y+3)(y^2 - 2y + 4) = y(y^2 - 2y + 4) + 3(y^2 - 2y + 4)$$
$$= y^3 - 2y^2 + 4y + 3y^2 - 6y + 12$$
$$= y^3 + y^2 - 2y + 12$$

2.

Problem:

Expand and simplify $(g-5)(g^2+3g-1)$

Solution:

$$(g-5) (g^2 + 3g - 1) = g (g^2 + 3g - 1) - 5 (g^2 + 3g - 1)$$
$$= g^3 + 3g^2 - g - 5g^2 - 15g + 5$$
$$= g^3 - 2g^2 - 16g + 5$$

Expanding double brackets practice

Go online



Q32: Expand and simplify $(b + 2)(b^2 - 4b + 3)$

.....

Q33: Expand and simplify $(g - 3)(2g^2 + 5g - 7)$

Expanding double brackets exercise

Go online



These questions are for practice only.

Q34: Expand and simplify (x-6)(x+3)

.....

Q35: Expand and simplify (y+3)(y-5)

......

Q36: Expand and simplify (t-2)(t-7)

.....

Q37: Expand and simplify (2x+6)(x-2)

.....

Q38: Expand and simplify (3r-1)(2r+5)

.....

Q39: Expand and simplify (w+x)(y+z)

.....

Q40: Expand and simplify (2y+4)(2y-4)

.....

Q41: Expand and simplify (x+5)(x-5)

.....

Q42: Expand and simplify $(x + 2)(x^2 + 3x - 5)$

.....

Q43: Expand and simplify $(a-3)(a^2-a-6)$

.....

Q44: Expand and simplify $(2f + 3)(3f^2 + 4f - 1)$

8.4 Learning points

When expanding a **single bracket** remember that every item inside the bracket is multiplied by the term outside the bracket.

When expanding **double brackets** remember that each term in the second bracket is multiplied by each term in the first bracket by turning the expression into two single bracket expressions or using the **rainbow method** or **FOIL**.

8.5 End of topic test

End of topic 8 test

Go online



Q45:

- a) Expand 3(x+1)
- b) Expand x(-2x-3)
- c) Expand 3y(2y-7)
- d) Expand 2y(5y+3z)
- e) Expand and simplify $6a(2a + 3b c^2)$
- f) Expand and simplify 3(b-4) + 4(b+3)
- g) Expand and simplify c(2c+3) + 2(3c-1)
- h) Expand and simplify d(3d+4) 5(d-2)

.....

Q46: Expand and simplify

- a) (w + x)(y + z)
- b) (x-3)(x-6)
- c) (x+1)(x-1)
- d) (2y-1)(y+3)
- e) (3y-4)(2y+3)
- f) $(z+4)(z^2+z-1)$
- g) $(z-2)(z^2+2z-5)$
- h) $(2x+5)(3x^2+2x-4)$

Answers to questions and activities

Topic 8: Expanding brackets

Answers from page 4.

Q1: 6x+24

Q2: 5x+7

Multiplying by a number practice (page 5)

Q3:

$$2(x - 1) = 2 \times x - 2 \times 1 = 2x - 2$$

Q4:

$$4(5x - 2) = 4 \times 5x - 4 \times 2$$
$$= 20x - 8$$

Multiplying by a letter practice (page 6)

Q5:

$$y(3y + 1) = y \times 3y + y \times 1$$
$$= 3y^2 + y$$

Q6:

$$y(3y + 5k) = y \times 3y + y \times 5k$$
$$= 3y^2 + 5ky$$

Multiplying by a number and a letter practice (page 7)

Q7:

$$7w(2w+3) = 7w \times 2w + 7w \times 3$$
$$= 14w^2 + 21w$$

Q8:

$$2m(3m^2 - 9n) = 2m \times 3m^2 + 2m \times (-9n)$$

= $6m^3 - 18mn$

Expanding single brackets practice (page 8)

Q9:

$$10(z+3) + 2(z-12) = 10 \times z + 10 \times 3 + 2 \times z + 2 \times (-12)$$
$$= 10z + 30 + 2z - 24$$
$$= 12z + 6$$

Q10:

$$3(f+4) - (f-3) = 3(f+4) - 1(f-3)$$

$$= 3 \times f + 3 \times 4 + (-1) \times f + (-1) \times (-3)$$

$$= 3f + 12 - f + 3$$

$$= 2f + 15$$

Remember that if there is no number in front of a bracket it means there is a 1. It is very important to put it in place.

Q11:

$$e(2e+8) - 2(e-5) = e \times 2e + e \times 8 + (-2) \times e + (-2) \times (-5)$$
$$= 2e^{2} + 8e - 2e + 10$$
$$= 2e^{2} + 6e + 10$$

Expanding single brackets exercise (page 8)

Q12: 2x - 2

Q13: -20x + 5

Q14: $2x^2 + 4x$

Q15: $-2x^2 + xy$

Q16: $10x^2 - 6x$

Q17: $12y^2 + 6yz$

Q18: $14xy + 21y^2 - 7yz$

Q19: 7x - 3

Q20: $2y^2 + 9y - 2$

Q21: 13z + 34

Multiplying double bracket expressions practice (page 11)

Q22:

$$(x+6)(x+4) = x^2 + 4x + 6x + 24$$
$$= x^2 + 10x + 24$$

Q23:

$$(a+3) (a-4) = a^2 - 4a + 3a - 12$$
$$= a^2 - a - 12$$

Q24:

$$(g-2)(g+3) = g^2 + 3g - 2g - 6$$

= $g^2 + g - 6$

Q25:

$$(k-1)(k-5) = k^2 - 5k - k + 5$$

= $k^2 - 6k + 5$

Q26:

$$(2x+1)(x+3) = 2x^2 + 6x + x + 3$$
$$= 2x^2 + 7x + 3$$

Q27:

$$(3h-2)(h+1) = 3h^2 + 3h - 2h - 2$$
$$= 3h^2 + h - 2$$

Q28:

$$(2m-3)(2m-1) = 4m^2 - 2m - 6m + 3$$
$$= 4m^2 - 8m + 3$$

Q29:

$$(4r+1)(2t-3) = 8t^2 - 12t + 2t - 3$$
$$= 8t^2 - 10t - 3$$

Difference of two squares practice (page 12)

Q30:

$$(b+4) (b-4) = b(b-4) + 4(b-4)$$
$$= b^2 - 4b + 4b - 16$$
$$= b^2 - 16$$

Q31:

$$(2g+7)(2g-7) = 2g(2g-7) + 7(2g-7)$$
$$= 4g^2 - 14g + 14g - 49$$
$$= 4g^2 - 49$$

Expanding double brackets practice (page 13)

Q32:

$$(b+2) (b^2 - 4b + 3) = b (b^2 - 4b + 3) + 2 (b^2 - 4b + 3)$$

= $b^3 - 4b^2 + 3b + 2b^2 - 8b + 6$
= $b^3 - 2b^2 - 5b + 6$

Q33:

$$(g-3) (2g^2 + 5g - 7) = g (2g^2 + 5g - 7) - 3 (2g^2 + 5g - 7)$$
$$= 2g^3 + 5g^2 - 7g - 6g^2 - 15g + 21$$
$$= 2g^3 - g^2 - 22g + 21$$

Expanding double brackets exercise (page 14)

Q34: $x^2 - 3x - 18$

Q35: $y^2 - 2y - 15$

Q36: $t^2 - 9t + 14$

Q37: $2x^2 + 2x - 12$

Q38: $6r^2 + 13r - 5$

Q39: wy + xy + wz + xz

Q40: $4y^2 - 16$

Q41: $x^2 - 25$

Q42: $x^3 + 5x^2 + x - 10$

Q43: $a^3 - 4a^2 - 3a + 18$

Q44: $6f^3 + 17f^2 + 10f - 3$

End of topic 8 test (page 16)

Q45:

a)
$$3x + 3$$

b)
$$-2x^2 - 3x$$

c)
$$6y^2 - 21y$$

d)
$$10y^2 + 6yz$$

e)
$$12a^2 + 18ab - 6ac^2$$

- f) 7b
- g) $2c^2 + 9c 2$
- h) $3d^2 d + 10$

Q46:

a)
$$wy + xy + wz + xz$$

b)
$$x^2 - 9x + 18$$

c)
$$x^2 - 1$$

d)
$$2y^2 + 5y - 3$$

e)
$$6y^2 + y - 12$$

f)
$$z^3 + 5z^2 + 3z - 4$$

g)
$$z^3 - 9z + 10$$

h)
$$6x^3 + 19x + 22x - 20$$