SCHOLAR Study Guide

National 5 Mathematics

Course Materials

Topic 14: Solving equations and inequations

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SCHOLAR Study Guide Course Materials Topic 14: National 5 Mathematics

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Topic 14

Solving equations and inequations

Contents

Learning objective

By the end of this topic, you should be able to:

- solve linear equations;
- interpret inequality symbols;
- solve linear inequations.

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14.1 Looking back at National 4: Solving simple equations and inequations

The next subtopics will help you remember how to solve simple equations, including those involving speed, distance and time, as well as solving simple inequations.

14.1.1 Solving simple equations

Solving Linear Equations Go online

First let us look at solving linear equations of the form $x + a = b$.

$x + 4 = 10\ldots$ What is x?

What plus 4 equals 10? 6

So, $x + 4 = 10\ldots$ $x = 6\ldots$ $(6 + 4 = 10)$

Now we will look at solving linear equations of the form $ax = b$.

$x \times 8 = 56\ldots$ What is x?

What times 8 equals 56? 7

So, $x \times 8 = 56\ldots$ $x = 7\ldots$ $(7 \times 8 = 56)$

Examples

1.

Problem:

Solve $x + 8 = 11$.

Solution:

Take 8 away from both sides of the equation, this will give you the value for x.

$x = 3$

. . . . . .

2.

Problem:

Solve $10x = 50$.

Solution:

Divide both sides by 10, this will give you the value for x.

$x = 5$

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Q1: Solve $x - 4 = 10$.

. . . . . .

Q2: Solve $7x = 42$.

14.1.2 Speed, distance and time

Speed, Distance and Time Go online

Let us look at the relationship between speed, distance and time.

A beetle walks along a ruler, covering 27 cm in 3 seconds. . . but not at an even pace.

The beetle travelled **27 cm** in **3 seconds** or **9 cm** in **1 second**.

We say that the beetle travelled at an ***average speed of 9 cm per second.***

Let's look at another example. . .

In a car race, the winner travels 450 miles in 3 hours.

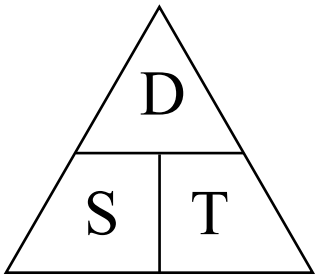
The car travelled **450 miles** in **3 hours** or **150 miles** in **1 hour**.

We say that the car travelled at an ***average speed of 150 miles per hour***.

Average Speed. . . Distance. . . Time

Given any two, you can calculate the third.

This triangle will help you. . .



Suppose you want the formula for Speed. . . cover up S to reveal $S = \frac{D}{T}$.

Or the formula for Distance. . . cover up D to reveal $D = S \times T$.

Or the formula for Time. . . cover up T to reveal $T = \frac{D}{S}$.

In general, given any two of speed, distance and time, we can work out the third.

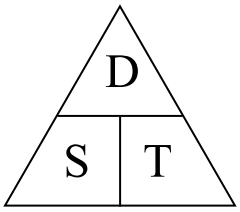
Examples

1.

Problem:

A cyclist travels at a speed of 35 miles per hour for 3 hours.
How far did he travel?

Solution:



$$D = S \times T$$

$$D = S \times T$$
$$D = 35 \times 3$$
$$D = 105 \text{ miles}$$

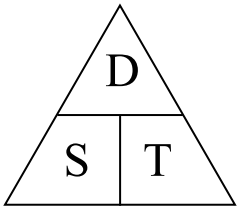
. . . . . .

2.

Problem:

A car covered 145 miles in 2·5 hours.
What was its average speed?

Solution:



$$S = D \div T$$

$$S = D \div T$$
$$S = 145 \div 2 \cdot 5$$
$$S = 58 \text{ miles / hour}$$

. . . . . .

3.

Problem:

A runner covered 26 miles at an average speed of 4 miles / hour.
How long did he take?

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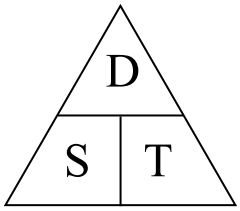
Solution:



$$T = D \div S$$

$$T = D \div S$$
$$T = 26 \div 4$$
$$T = 6 \cdot 5 \text{ hours}$$

Q3: A driver travels at a speed of 45 miles per hour for 2 hours. How far did they travel?

. . . . . .

Q4: A walker travels 7·5 miles for 2·5 hours. What was their average speed?

. . . . . .

Q5: A cyclist covered 130 miles at an average speed of 26 miles / hour. How long did it take them?

14.1.3 Solving simple inequations

Solving Inequalities Go online

Let us look at solving inequalities of the form $x + a > b$.

If we have a statement like:

$x + 4 > 11$ and it is true. . .

Then adding or subtracting 1 on both sides will change it:

$$x + (4 + 1) > (11 + 1)$$
$$x + 5 > 12$$

Or

$$x + (4 - 1) > (11 - 1)$$
$$x + 3 > 10$$

Both equations are still true!

Now we will look at solving inequalities of the form $ax < b$.

If we have a statement like:

$8x < 24$ and it is true. . .

Then dividing both sides by the same number will change it:

$$\left(\frac{8}{2}\right)x < \left(\frac{24}{2}\right)$$
$$4x < 12$$

But it will still be true.

If we divide both sides by 8 we find that $x < 3$:

$$\left(\frac{8}{8}\right)x < \left(\frac{24}{8}\right)$$
$$x < 3$$

We call this the solution to the inequality.

Examples

1.

Problem:

Solve the inequality $x + 5 > 12$.

Solution:

$x + 5 > 12$
Subtracting 5 from both sides gives:
$x > 7$

. . . . . .

2.

Problem:

Solve the inequality $6y < 48$.

Solution:

$6y < 48$
Dividing both sides by 6 gives:
$y < 8$

Q6: Solve the inequality $x + 4 > 22$.

. . . . . .

Q7: Solve the inequality $4y < 36$.

14.2 Solving linear equations

When solving equations we are trying to find the value of the variable. x is a **variable**, some other letters which are commonly used instead of x are d, n, p, t, y and z.

A linear equation takes the form $ax + b = c$ where a, b and c are **constants** and $a \neq 0$. You will already know how to solve equations like $3x = 15$ and $x - 5 = 13$ so you already know how to solve simple linear equations.

There are several methods that you may already have used such as the cover-up, balancing and change side change sign.

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In this topic we are going to learn how to solve more complex linear equations.

To solve an equation we are trying to find the value of the variable which makes the equation true.

Examples

1.

Problem:

Solve $5x - 4 = 26$

Solution:

$$\begin{aligned} 5x - 4 &= 26 && \text{get rid of the } -4 \text{ from the left by } +4 \\ 5x - 4 + 4 &= 26 + 4 \\ 5x &= 30 \\ 5x \div 5 &= 30 \div 5 && \text{leave } x \text{ on its own by } \div 5 \\ x &= 6 \end{aligned}$$

Note: We don't have to show the lines of working to remove the -4 and find x on its own by $\div 5$.

If we substitute $x = 6$ into the equation we will see that it makes the equation true.

$$\begin{aligned} 5x - 4 &= 26 \\ 5 \times 6 - 4 &= 26 \\ 30 - 4 &= 26 \\ 26 &= 26 \end{aligned}$$

Remember to underline your solution.

. . . . . .

2.

Problem:

Solve $3y - 6 = y - 10$

Solution:

Notice that there are y terms on both sides of the equation.
Our aim should always be to collect the letters on one side of the equation and the numbers on the other.

$$\begin{aligned} 3y - 6 &= y - 10 && \text{get rid of } y \text{ from the right by } -y \\ 3y - y - 6 &= y - y - 10 \\ 2y - 6 &= -10 && \text{get rid of } -6 \text{ from the left by } +6 \\ 2y - 6 + 6 &= -10 + 6 \\ 2y &= -4 && \text{leave } y \text{ on its own by } \div 2 \\ y &= -2 \end{aligned}$$

. . . . . .

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3.

Problem:

Solve $3(a + 2) = 18$

Solution:

$3(a+2) = 18$ expand the brackets

$3a + 6 = 18$ get rid of $+6$ from the left by -6

$3a = 12$ leave a on its own by $\div 3$

$a = 4$

. . . . . .

4.

Problem:

Solve $2b + 7(b + 2) = 4 - b$

Solution:

$2b + 7(b+2) = 4 - b$ expand the brackets

$2b + 7b + 14 = 4 - b$ collect like terms

$9b + 14 = 4 - b$ get rid of $-b$ from the right by $+b$

$10b + 14 = 4$ get rid of $+14$ from the left by -14

$10b = -10$ leave b on its own by $\div 10$

$b = -1$

. . . . . .

5.

Problem:

Solve $3d - 4(d - 1) = 7(4 - d)$

Solution:

$3d - 4(d-1) = 28 - 7d$ expand like brackets

$3d - 4d + 4 = 28 - 7d$ collect like terms

$-d + 4 = 28 - 7d$ get rid of $-7d$ from the right by $+7d$

$6d + 4 = 28$ get rid of $+4$ from the left by -4

$6d = 24$ leave d on its own by $\div 6$

$d = 4$

. . . . . .

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6.

Problem:

Solve $50 - (f - 5) = 2(f + 11)$

Solution:

$50 - (f - 5) = 2(f + 11)$ expand the brackets

$50 - f + 5 = 2f + 22$ collect like terms

$-f + 55 = 2f + 22$ get rid of $+2f$ from the right by $-2f$

$-3f + 55 = 22$ get rid of $+55$ from the left by -55

$-3f = -33$ leave f on its own by $\div -3$

$f = 11$

. . . . . .

7.

Problem:

Solve $^2/_3\, g + 4 = 6$

Solution:

Notice that each term in the equation must be multiplied by 3 so that the meaning of the equation is not changed.

$\frac{2}{3}g + 4 = 6$ remove the fraction by $\times 3$

$3 \times \frac{2}{3}g + 3 \times 4 = 3 \times 6$

$\frac{6}{3}g + 12 = 18$ simplify $\frac{6}{3} = 2$

$2g + 12 = 18$ get rid of $+12$ by -12

$2g = 6$ leave g on its own by $\div 2$

$g = 3$

. . . . . .

8.

Problem:

Solve $^1/_4\,(8p - 2) = 5$

Solution:

$\frac{1}{4}(8p - 2) = 5$ expand the brackets by $\div 4$

$2p - \frac{1}{2} = 5$

$2 \times 2p - 2 \times \frac{1}{2} = 2 \times 5$ remove the fraction by $\times 2$

$4p - 1 = 10$ get rid of -1 by $+1$

$4p = 11$ leave p on its own by $\div 4$

$p = \frac{11}{4}$

Notice that the answer is an improper fraction.
You can leave it like this or change it into a mixed number $p = 2\,^3/_4$.

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Solving linear equations practice

Go online

Q8: Solve $8z + 4 = -20$

.....

Q9: Solve $5 - 2t - 2 = t$

.....

Q10: Solve $4(2r + 1) = 20$

.....

Q11: Solve $5(6 - m) + 2 = m + 2$

.....

Q12: Solve $2(3k + 5) = 3k - 4(k + 1)$

.....

Q13: Solve $-2(2n + 1) = 3 - (n - 1)$

.....

Q14: Solve $^3/_4\, j + 5 = 2$

Solving linear equations exercise

Go online

Solving linear equations

Q15: Solve:

a) $6a - 7 = 23$
b) $4b + 2 = b - 10$
c) $5c + 7 = 8c - 2$

Solving linear equations with brackets

Q16: Solve:

a) $3(d + 2) = 21$
b) $2(4 - e) = 4e - 10$
c) $5(2f - 1) = 8(f + 2)$
d) $g - 3(g - 2) = 3(2 - g)$
e) $6(h - 1) - 5 = 4 - (h + 1)$

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Solving linear equations with fractions

Q17: Solve:

a) $^1/_4\, j + 3 = 2$
b) $^3/_5\, k - 1 = 2$
c) $^2/_3\, m + {}^5/_3 = 7$
d) $^1/_2\, (n + 14) = 6$
e) $^5/_8\, p + 3 = 5$

14.3 Solving linear inequations

An inequality or inequation defines the relationship between two quantities. We know the equality sign '=' which we have already seen in equations. The four signs that appear in inequations are $>$, $\geq$, $<$ and $\leq$.

- $>$ means 'is greater than'
- $\geq$ means 'is greater than or equal to'
- $<$ means 'is less than'
- $\leq$ means 'is less than or equal to'

When the inequality involves a **variable** then its solution is the set of values that make the statement true.

We can solve inequations, sometimes called inequalities, in a similar way to equations.

Examples

1.

Problem:

Solve $5x + 11 > 26$

Solution:

$$5x + 11 > 26$$ get rid of the $+11$ from theleft by -11
$$5x + 11 - 11 > 26 - 11$$
$$5x > 15$$ leave x on its own by $\div 5$
$$5x \div 5 > 15 \div 5$$
$$x > 3$$

The solution to this inequality is: x is greater than 3.
Possible values for x are 4, 11, 12·3, 25 $^1/_2$, 100, . . .

Note: We don't have to show the lines of working to remove the $+11$ and $\div 5$.

. . . . . .

2.

Problem:

Solve $6y + 7 \leq y - 28$

Solution:

$6y + 7 \leqslant y - 28$ get rid of the y from the right by $-y$

$5y + 7 \leqslant -28$ get rid of the $+7$ from the left by -7

$5y \leqslant -35$ leave y on its own by $\div 5$

$y \leqslant -7$

. . . . . .

3.

Problem:

Solve $3a + 8 < 26$

Solution:

$-3a + 8 < 26$ get rid of $+8$ from the left by -8

$-3a < 18$ leave a on its own by $\div -3$

Note: When we divide by a negative we must reverse the inequality sign

$-3a \div -3 > 18 \div -3$

$a > -6$

Key point

Consider $8 > 6$.

Dividing both sides by -2 gives $-4 > -3$ but this is no longer true.

We must reverse the inequality sign to make this statement true, giving: $-4 < -3$

Similarly when we multiply on both sides of an inequality by a negative we must reverse the inequality sign.

Examples

1.

Problem:

Solve $10 - 2(d + 3) < d + 19$

Solution:

$10 - 2(d + 3) < d + 19$ expand the brackets

$10 - 2d - 6 < d + 19$ collect like terms

$-2d + 4 < d + 19$ get rid of d from the right by $-d$

$-3d < 15$ get rid of $+4$ from the left by -4

$d > -5$

. . . . . .

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2.

Problem:

Solve $^1/_2t \ - \ 2 < 5 \ + \ t$

Solution:

$\frac{1}{2}t - 2 < 5 + t$ remove the fraction by $\times 2$

$t - 8 < 10 + 2t$ get rid of $+ 2t$ by $- 2t$

$-t - 8 < 10$ get rid of $- 8$ by $+ 8$

$-t < 18$ leave t on its own by $\div -1$ and reverse the sign

$t > 18$

. . . . . .

3.

Problem:

A gym offers a monthly membership for £32. As a member the cost for each class attended is £1. It is also possible for non-members to drop-in to classes. The drop-in cost for a class is £5.

Let n be the number of classes attended in a month.

a) Construct an expression for the total cost for a member to attend n classes.

b) Construct an expression for the total cost for a non-member to attend n classes.

c) Set up an inequation and solve it to find the minimum number of classes that must be attended each month before taking out a membership is the cheaper option.
Explain your answer.

Solution:

a) Members would pay $£32 \ + \ n \ \times \ £1 \ = \ 32 \ + \ n$

b) Non-members would pay $n \ \times \ £5 \ = \ 5n$

c) We could check to see when non-members pay more than members.

$5n \ > \ 32 \ + \ n$

$4n \ > \ 32$

$n \ > \ 8$

It is best to try a value for n to explain your answer.

When $n \ = \ 9$ members pay $32 \ + \ 9 \ = \ £41$

When $n \ = \ 9$ non-members pay $5 \ \times \ 9 \ = \ £45$

When 9 or more classes are attended each month it is cheaper to be a member.

Key point

Remember: when we solve an inequation by multiplying or dividing by a negative we must reverse the inequality sign.

Solving inequations practice Go online

Q18: Solve $3z + 4 < -20$

.....

Q19: Solve $10z - 23 \geq 3z + 26$

.....

Q20: Solve $5 - 2b \leq 19$

.....

Q21: Solve $2g - 7(g + 2) \geq 3(2 - g)$

Solving inequations exercise Go online

Solving simple inequations

Q22: Solve:

a) $12a + 7 > 31$
b) $5b - 9 \leq b + 19$
c) $2(3c + 7) \geq 2(2 - 2c)$
d) $15 - (d + 3) < 2 - 2d$
e) $11e - 4(2e - 5) \leq e + 12$

Solving more complex inequations

Q23: Solve:

a) $5 - 2f < -9$
b) $-6g + 8 > 2g - 8$
c) $5(1 - 2h) \leq 11 - 4h$
d) $^1/_2\,(6 - j) \geq 22 - j$
e) $2(15 - k) < {}^1/_2\,k$

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14.4 Learning points

Solving linear equations

- Show every step as a new line in your solution.
- To get rid of a term to the other side of an equation carry out the inverse operation:
  - add becomes subtract;
  - subtract becomes add;
  - multiply becomes divide;
  - divide becomes multiply;
- If the equation has brackets multiply them out first.
- If the equation has a fraction, multiply every term in the equation by the denominator (or by the common denominator if there is more than one fraction).

Solving inequations

- An inequation will contain $>$, $\geq$, $<$ or $\leq$.
  - $>$ means 'is greater than'
  - $\geq$ means 'is greater than or equal to'
  - $<$ means 'is less than'
  - $\leq$ means 'is less than or equal to'
- Reverse the inequality when dividing or multiplying by a negative.

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14.5 End of topic test

End of topic 14 test Go online

Solving Linear Equations

Q24: Solve:

a) $7x + 5 = 3x - 7$
b) $6(x - 3) + 8 = 2(3 - x)$
c) $^2/_5\, x + 9 = 7$

Solving Inequations

Q25: Solve:

a) $8x + 4 \geq 5x - 8$
b) $2x + 4 > 5x - 14$
c) $x + 5 < {}^1/_2(4x - 10)$

Glossary

constants

a mathematical constant is a numerical value or symbol which is fixed e.g. 5, 2·075, -1000, π, ...

variable

a mathematical variable is a symbol for a number we do not know yet, it is usually a letter like x or y

Answers to questions and activities

Topic 14: Solving equations and inequations

Answers from page 4.

Q1: 14

Q2: 6

Answers from page 6.

Q3: 90

Q4: 3

Q5: 5

Answers from page 7.

Q6:

$x + 4 > 22$
Subtracting 4 from both sides gives:
$x > 18$

Q7:

$4y < 36$
Dividing both sides by 4 gives:
$y < 9$

Solving linear equations practice (page 11)

Q8:

Steps:

- $8z = -24$

Answer: $z = -3$

Q9:

Steps:

- $-2t + 3 = t$
- $-3t + 3 = 0$
- $-3t = -3$

Answer: $t = 1$

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Q10:

Steps:

- $8r + 4 = 20$
- $8r = 16$

Answer: $r = 2$

Q11:

Steps:

- $30 - 5m + 2 = m + 2$
- $-5m + 32 = m + 2$
- $-6m + 32 = 2$
- $-6m = -30$

Answer: $m = 5$

Q12:

Steps:

- $6k + 10 = 3k - 4k - 4$
- $6k + 10 = -k - 4$
- $7k + 10 = -4$
- $7k = -14$

Answer: $k = -2$

Q13:

Steps:

- $-4n - 2 = 3 - n + 1$
- $-3n - 2 = 4$
- $-3n = 6$

Answer: $n = -2$

Q14:

Steps:

- $3j + 20 = 8$
- $3j = -12$

Answer: $j = -4$

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Solving linear equations exercise (page 11)

Q15:

a) $a = 5$
b) $b = -4$
c) $c = 3$

Q16:

a) $d = 5$
b) $e = 3$
c) $f = 10 \cdot 5$
d) $g = 0$
e) $h = 2$

Q17:

a) $j = -4$
b) $k = 5$
c) $m = 8$
d) $n = -2$
e) $p = 3.2$ or $^{16}/_{5}$ or $3^{1}/_{5}$

Solving inequations practice (page 15)

Q18:

Steps:

- $3z < -24$

Answer: $z < -8$

Q19:

Steps:

- $7z - 23 \geq 26$
- $7z \geq 49$

Answer: $z \geq 7$

Q20:

Steps:

- $-2b \leq 14$

Answer: $b \geq -7$

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Q21:

Steps:

- $2g - 7g - 14 \geq 6 - 3g$
- $-5g - 14 \geq 6 - 3g$
- $-2g - 14 \geq 6$
- $-2g \geq 20$

Answer: $g \geq -10$

Solving inequations exercise (page 15)

Q22:

a) $a > 2$
b) $b \leq 7$
c) $c \geq -1$
d) $d < -10$
e) $e \leq -4$

Q23:

a) $f > 7$
b) $g < 2$
c) $h \geq -1$
d) $j \geq 38$
e) $k > 12$

End of topic 14 test (page 17)

Q24:

a) $x = -3$
b) $x = 2$
c) $x = -5$

Q25:

a) $x \geq -4$
b) $x < 6$
c) $x > 10$

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